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"The Power of Money: Wealth Effects in Contest"

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The Power of Money: Wealth Effects in Contests*

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Abstract

The relationship between wealth and power has long been debated. Nevertheless, this relationship has been rarely studied in a strategic game. In this paper, we study wealth effects in a strategic contest game. Two opposing effects arise: wealth reduces the marginal cost of effort but it also reduces the marginal benefit of winning the contest. We consider three types of contests which vary depending on whether rents and efforts are commensurable with wealth. Our theoretical analysis shows that the effects of wealth are strongly "contest-dependent". It thus does not support general claims that the rich lobby more or that low economic growth and wealth inequality spur conflicts.

Keywords: Conflict, contest, rent-seeking, wealth, risk aversion, lobbying, power, redistribution.
JEL codes: C72, D72.

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"Pecunia nervus belli."

1 Introduction

1.1 The general motivation

As popularized by Frank and Cook's (1995) best-selling book "The Winner-Take-All Society" many competitive situations take the form of a contest. Examples include political lobbying, research and development, marketing, promotion, status-seeking, and litigation activities (Konrad 2009). In this paper, we are interested in the effect of wealth in contests. In particular, the motivation for our analysis includes general questions such as: Do rich people lobby more? Does poverty lead to more conflicts? Does low economic growth and wealth inequality increase political activism?

The relationship between wealth and power has attracted attention for centuries (Marx 1867, Wright Mills 1956). The conventional wisdom suggests that the rich are more powerful than the poor.[1] Bartels (2005) and Gilens (2005) observe, for instance, that US senators are more responsive to the opinions of their more affluent constituents. This idea has been extensively discussed in political science, often pejoratively referred as plutocracy. It is exemplified by Hacker and Pierson's (2011) recent book "Winner-Take-All Politics: How Washington Made the Rich Richer–and Turned Its Back on the Middle Class".

Nevertheless, in contrast, casual observation suggests that low wealth induces greater participation and effort in contest-type situations. People involved in highly predatory and competitive activities, such as thieves or athletes for instance, typically come from poorer segments of society. More

[1] This is consistent with the beliefs of some prominent economists. For instance, Anne Krueger (1974), in her pioneering work on rent seeking argues that we can perceive the price system "as a mechanism rewarding the rich and well-connected". Likewise Jack Hirshleifer (1995) stresses that "the half of the population above the median wealth surely has greater political strength than the half below". Paul Krugman (2010) similarly observes that "the rich are different from you and me: they have more influence". Lastly, Daron Acemoglu (2013) declares that "the rise in inequality has created a class of very wealthy citizens who can use their wealth to gain more political power — partly to defend their wealth and partly to further their economic, political, and ideological agendas".

corruption is also typically observed in poorer countries (Aidt 2009, Gundlach and Paldam 2009). Some groups (e.g., farmers), although often relatively poor, are well-known to be politically powerful. Poverty has also been found to be a robust factor in explaining violent crime and civil conflicts (Collier and Hoeffler 1998, Fajnzylberg, Lederman and Loayza 2002, Fearon and Laitin 2003, Blattman and Miguel 2010). Relatedly, it is often said that redistribution policies favour political stability and social peace. Finally, a recent experimental study shows that wealth has non-monotonic effects on conflict intensity (Baik, Chowdhury and Ramalingam 2015).

1.2 The model, and the basic effects

Although the above observations concern disparate issues, they suggest that wealth may have fundamentally different, and perhaps opposing, effects in contests. This further suggests that economic theory may help us pin down and examine the strength of these effects. Yet, wealth effects have received little attention in the (otherwise vast) theoretical literature on contests (Tullock 1980, Garfinkel and Skaperdas 2007, Konrad 2009, Congleton, Hillman and Konrad 2010). Indeed, the "workhorse" model in this literature, often coined the Tullock contest model, is based on a strategic game in which wealth plays essentially no role. In this game, each agent has the following payoff function:

$$U_i = w_i - x_i + \Pi_i r, \tag{1}$$

in which x_i is agent i's effort, $r > 0$ is the rent (i.e., the prize) for the contest winner and Π_i is the probability of winning the contest. Notice immediately then that individual wealth w_i enters separately in the payoff function (1), and thus has no effect on the agent's effort (hence, wealth is, without loss of generality, normalized to zero in the literature).

In this paper, we adapt this basic contest model in order to examine the effects of wealth. Namely, we introduce in model (1) a utility function $u(\cdot)$ that displays the familiar property of decreasing marginal utility of wealth, i.e. $u'' < 0$. Note immediately that this introduction requires to specify whether the rent r and efforts x_i can be expressed in monetary terms, and thus are commensurable with wealth w_i within the utility function. This specification is central. Indeed, it technically removes the separability of wealth with the rent and efforts in model (1). Moreover, it permits to pin down two basic effects that we believe should naturally arise in contests:

- First, wealth can reduce the marginal cost of effort. To illustrate, note that it is marginally less costly for a rich person than a poor person to offer a monetary payment to, e.g., a politician, in order to obtain some privilege. The rich can thus relatively more easily afford costly expenditures in a contest than the poor, other things being equal.

- Second, and in contrast to the first effect, wealth may decrease the marginal benefit of winning a contest. To illustrate, note that it is marginally more beneficial for the poor to obtain the monetary reward associated with victory in a contest. We may thus regard the poor as being relatively more motivated to exert effort in a contest than the rich, other things being equal.

Although these effects are simple, their analysis is not trivial because of strategic considerations. If a change in wealth affects the level of effort of one player, the other player is expected to react to this change, which in turn affects the initial player and so on. There is thus a need to carefully examine the overall impact of wealth on the players' equilibrium efforts using the standard tools of game theory.

1.3 The contest success function

In strategic contest games, the probability Π_i is usually coined the contest success function (CSF). This function defines the contest "technology" and it strongly affects the properties of the game. In our two-player game ($i = a, b$), we will often denote the probability of a winning as $p(x_a, x_b)$ such that $\Pi_b = 1 - p(x_a, x_b)$. We will assume throughout that the CSF has the power-logistic form (Tullock 1980):[2] i.e.,

$$\Pi_a = p(x_a, x_b) = \frac{x_a^m}{x_a^m + x_b^m}, \tag{2}$$

[2]This CSF is a special case of logistic functions, $p(x_a, x_b) = \frac{\Phi(x_a)}{\Phi(x_a)+\Phi(x_b)}$. Garfinkel and Skaperdas (2007) and Konrad (2009) discuss the axiomatic foundations and economic illustrations for this special, but common, class of CSFs. Moreover, Jia (2008) shows that this CSF can also be motivated as the probability of winning a rank-order tournament (as in Lazear and Rosen, 1981), when the noise terms are drawn from the inverse exponential distribution (see also Jia *et al.*, 2013). Under these conditions, the contests that we consider may therefore also be considered as rank-order tournaments.

with $x_i > 0$ $(i = a, b)$ and $m > 0$. If a exerts twice as much effort as b, the odds that he wins over b is $2^m : 1$. The parameter m is thus the contest decisiveness parameter measuring how important relative effort $(\frac{x_i}{x_j})$ is compared to random factors for winning the contest (Hirshleifer 1991). If $m \to 0$, each player wins the contest with probability $\frac{1}{2}$ independently of the levels of effort. The larger m, the more pronounced the effects of relative differences in effort on the likelihood of winning.

While this CSF is increasing and homogeneous of degree zero in its arguments, concavity is only guaranteed for arbitrary effort levels when $m \leq 1$. Because of the salient use of the "workhorse" model (1) in the literature, the assumption $m \leq 1$ is prevalent (an exception is Perez-Castrillo and Verdier 1992). From an empirical stance, this assumption seem unduly restrictive.[3] There are typically two ways to relax this assumption. The first is by restricting oneself to symmetric contests. As shown by Baye, Kovenock and De Vries (1994) in a symmetric Tullock contest, a symmetric Nash equilibrium in pure strategies exists as long as $m \leq 2$.[4] Since we are interested in the effects of changes in wealth and in wealth inequality, the symmetry assumption is not appropriate. A second way out is to have sufficiently increasing marginal disutility of effort. Indeed, in our contest models, players' preferences are nonlinear in efforts. Hence we need not restrict m to be in the unit interval, and all our numerical examples consider $m > 1$.

The properties of $p(\cdot)$ are given in Appendix A.2. Here, we draw attention to the following property

$$p_{12} = \frac{\partial^2 p}{\partial x_a \partial x_b} = \frac{m^2}{x_a x_b} \Pi_a \Pi_b (\Pi_a - \Pi_b), \tag{3}$$

meaning that *the marginal productivity of one player's effort is enhanced by the other player's effort iff the former exerts more effort.* Thus, the strategic models we will consider are neither games of strategic complements nor of

[3] To illustrate, Hwang (2009) provides an idea about the order of magnitude of m in some contests. Using data from battles fought in 17th century Europe and during World War II, he obtains values of .704 (.120) and 3.420 (.678), respectively (standard errors in brackets).

[4] For larger m, the first- and second-order conditions fail to characterise the best-response functions, the reason being that a single player can profitably deviate from the (symmetric) solution to these conditions by exerting zero effort.

strategic substitutes. In fact, as in Nti (1999) and Acemoglu and Jensen (2013) for instance, some interesting features arise in our contest models because the change in the effort of one player will either increase the effort of the other player (when this player wants to "keep up") or decrease this effort (because this other player "gives up").

1.4 The related literature

A few studies have discussed the effect of wealth in strategic models of contests. These effects differ significantly from those considered in this paper. We first discuss Hirshleifer's (1991) so-called "paradox of power". In its weak form, this paradox states that the final distribution of wealth will have less dispersion than the initial distribution of wealth. In its strong form, it states that there should be equal final distribution of wealth. Hirshleifer (1991) presents numerical examples in a contest model for which the paradox of power does, or does not, hold. In his model, initial wealth is divided between productive effort and fighting effort. The key difference to our model is thus that the rent, i.e., aggregate production, is endogenous.

A related model is the "winner take all with limited liability" model introduced in Skaperdas and Gan (1995). The agent's payoff in this model essentially writes as follows

$$U_i = \Pi_i u(w_i - x_i).$$

An interpretation of this model is that the loser "dies" and obtains utility $u(0) = 0$. Although Skaperdas and Gan (1995) does not study the effect of wealth (but that of risk aversion), it is easy to understand that wealth has a positive effect in this model. The intuition is that the two effects we have identified above go in the same direction. Wealth decreases the marginal cost of effort (as described above), but wealth also increases the marginal benefit of effort. Indeed, this last effect simply means that life is more valuable when wealthier.

Che and Gale (1997) examine the effect of budget constraints in a basic contest model as in (1). They show that each agent's equilibrium effort is a weakly increasing function of the agent's budget constraint, and that the presence of budget constraints lowers aggregate effort. Therefore, if one naturally assumes that a wealthier agent is less budget-constrained, wealth has a positive effect on effort. The effect of a budget constraint can be interpreted as an extreme utility curvature of the utility function $u(\cdot)$ at zero. However, since the utility function is otherwise linear, there are no wealth effects when wealth changes occur within the bounds of unconstrained efforts. Moreover, the model does not capture the effect that wealthier agents may have a lower marginal benefit of obtaining rent.

Finally, we briefly discuss Grossman's (1991) model of insurrection. This model sensitively differs from model (1), but has been quite influential in the conflict literature (Azam 2006, Chassang and Padro-i-Miquel 2009, Besley and Persson 2012). Grossman (1991) considers a general equilibrium model in which agents choose how to devote their time between production, soldiering (for the government) and insurrection. This implies that income (generated by production) and conflict expenditures are endogenously determined in equilibrium. The more decisive is the CSF, the larger is the fraction of time devoted to insurrection as opposed to production. As a result, there is an equilibrium associating low wealth and high conflict expenditures. A key insight from this model is that participation in soldiering increases with the opportunity cost of fighting.

1.5 The organization of the paper

As we said above, two wealth effects typically arise in contests: i) wealth decreases the marginal cost of effort, but also ii) decreases the marginal benefit of winning the contest. We study in turn three strategic contest models depending on whether the first, second, or both wealth effects play a role. In each of these models, we compare the levels of efforts of a poor and a rich within an equilibrium. We also compare these levels across equilibria that differ only by a change in the wealth of one or both players. We characterize wealth effects both on individual efforts and on aggregate efforts, and also study the effect of a wealth transfer among the two players.

Since we use the comparative statics method, it is important to stress that we will focus on the part of the parameter space for which a unique pure strategy Nash equilibrium exists. The parameter space is made up of the two wealth levels, the CSF decisiveness parameter, the size of the prize and the curvature parameters of the players' utility and cost functions. The main text presents and discusses the different models and wealth effects, and only provides a sketch of the results together with some graphical illustrations. All the results are demonstrated in the Appendix.

We consider in Section 2 the so-called "privilege contest" model. In this model, effort is monetary, but the rent —i.e., the privilege— is non-monetary and therefore its marginal value is independent of the level of wealth. Therefore, only the first effect on marginal cost described above is active. We then consider in Section 3 a model in which only the second effect described above on the marginal benefit is active, the so-called "ability contest" model. In this alternative model, rent is monetary but effort —which determines ability— is non-monetary and so the marginal cost of effort is independent of wealth. We show that the effect of increasing wealth on agent effort is positive in the privilege contest model while it is negative in the ability contest model. We also examine the effect of wealth redistribution in both models, and find that this effect tends to decrease aggregate effort for a low decisiveness parameter.

In Section 4, we study a model in which both the rent and the efforts are monetary, the so-called "rent-seeking contest" model, corresponding to the rent-seeking model with risk aversion (Cornes and Hartley 2012). In this model, we show that under constant absolute risk aversion (CARA), the two opposing wealth effects exactly offset each other so that wealth has no effect on the efforts of agents. Moreover, we show that wealth tends to increase effort under a specific condition on the utility. This condition appeared in the single-agent risk theory literature (Eeckhoudt, Gollier and Schlesinger 1996), and is such that more background risk increases risk aversion. We demonstrate that under this condition, a rich agent exerts relatively more effort than a poor agent, and that an isolated increase in the wealth of the rich agent always increases that agent's effort, but reduces the effort of the poor agent. Finally, Section 5 summarizes all the wealth effects in the privilege, ability and rent-seeking contest models, and concludes our analysis.

The appendix provides the technicalities supporting the analysis presented in the main text. In Appendix A.1, using a single-crossing property, we provide a general comparative statics method to study the effects of wealth in a two-player game. In Appendix A.2, we display the properties of the CSF. In Appendix A.3, we characterize some conditions ensuring existence, uniqueness and stability. In Appendices A.4, A.5 and A.6, we respectively demonstrate the results for the privilege, ability and rent-seeking contest models. In Appendix A.7, we provide demonstrations of the results regarding the effects of wealth transfers in each contest model. These last results are the most complex to derive, and we often restrict the analysis to CARA, quadratic or constant relative risk aversion (CRRA) utility functions.

2 The privilege contest model

In the privilege contest model, the rent is non-monetary. Our chief interpretation is that the benefit from winning the contest is only associated with a form of prestige (or "ego-utility"), without any financial counterpart. This model of contest may include, for instance, status-seeking activities, political campaign contributions or warfare for purely ideological motives.

Denoting the non-monetary benefit of the privilege as r, we model the preferences of player i $(= a, b)$ with wealth w_i and exerting effort x_i as

$$U_i = u(w_i - x_i) + \Pi_i r. \tag{4}$$

Observe that in this model wealth w_i and effort x_i are commensurable within the utility function $u(\cdot)$. In the rest of the paper, we will assume that $u(\cdot)$ is thrice continuously differentiable with $u' > 0$ and $u'' < 0$. The marginal utility of effort,

$$\frac{\partial U_i}{\partial x_i} = -u'(w_i - x_i) + \frac{\partial \Pi_i}{\partial x_i} r,$$

reveals that the marginal benefit of effort will depend on x_j $(j \neq i)$ in a way consistent with (3), while the marginal cost of effort (the first *rhs* term) does not.

We start our analysis with the simpler symmetric case, $w_a = w_b = w$. Using (2), the equilibrium condition for the symmetric equilibrium x is given by

$$H_P(w) \stackrel{\text{def}}{=} -u'(w - x) + \frac{m}{4x} r = 0.$$

This condition captures the tension between the marginal cost and the marginal benefit of effort. Observe that wealth only affects the term $-u'(w-x)$, and that this effect is positive. Indeed, the effect of wealth is given by the sign of $H'_P(w) = -u''(w - x) > 0$. This represents the first effect mentioned above that wealth reduces the marginal cost of effort. Thus in the privilege model, effort is *a normal good*.

The non-symmetric equilibrium case is more complex, but the intuition is easy to grasp by looking at "first round" effects. Let $w_a > w_b$. Effort being a normal good, an increase in the wealth of the poorer agent, w_b, will trigger an increase in x_b. Since x_a exceeds x_b in this case, the increase in x_b will raise the marginal benefit of the rich agent's effort as given by (3), thereby triggering an increase in that effort, and thus also in total effort. On the other hand, when the rich agent a gets wealthier, he will adjust his effort up, which in view of (3) will lower the marginal benefit of his poorer opponent, who reacts by adjusting her effort downwards. The last mechanism is particularly strong if the effort levels, and thus the wealth levels, are far apart, so that it may lead to a reduction in total effort. A proper comparative statics analysis complements these first round effects with feedback effects, and is presented fully in the Appendix A.4. Here in Figure 1, we further illustrate this analysis by drawing best-response functions for players a and b, denoted $f(x_b, w_a)$ and $g(x_a, w_b)$ respectively.

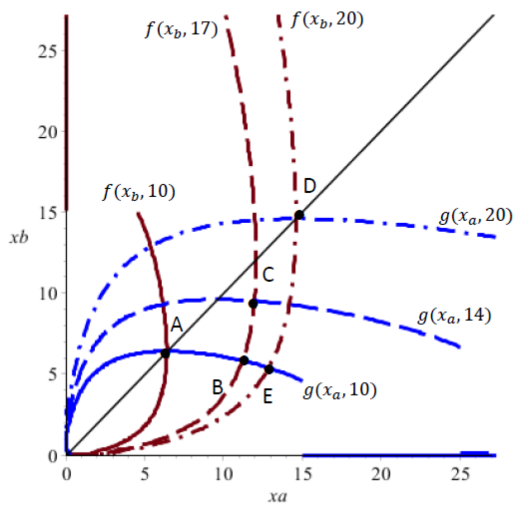

Figure 1. Equilibria in the privilege contest model for different wealth combinations.

In this Figure, we assume that $u(w) = \frac{w^{1-\rho}}{1-\rho}$ (i.e., CRRA) with $\rho = 2$, $m = 2$, $r = 1$. The best-response functions of players a and b are drawn for $w_a = 10, 17$ and 20, and for $w_b = 10, 14$, and 20. First observe that for large efforts of the opponent, the best-response for each player is zero effort, hence the discontinuity in the best-response functions (visible in the figure for w_a and w_b equal to 10). Note that these functions are first increasing and then decreasing, with a maximum at $x_a = x_b$. Note also that an increase in w_a moves the best-response function $f(x_b, w_a)$ rightward and an increase in w_b moves the best-response function $g(x_b, w_b)$ upward. Simple wealth effects naturally follow. Point A represents a symmetric equilibrium with uniform low wealth ($w_a = w_b = 10$), while point D represents a symmetric equilibrium with uniform high wealth ($w_a = w_b = 20$). A move from A to D thus illustrates that efforts increase with a common increase in wealth, as shown theoretically above. Observe also a within-equilibrium result: wealthier agents exert more efforts. Indeed, point B represents for instance an equilibrium with $w_a = 17 > w_b = 10$ leading to $x_a > x_b$. The move from B to C illustrates the effect of an increase in w_b from 10 to 14 leading to increase in both x_a and x_b. Point E is the result of an increase in w_a from 17 to 20 leading to an increase in x_a but a decrease in x_b. Thus, total effort grows with an increase in w_b but not necessarily with an increase in w_a (see the Appendix A.4 for a sufficient condition to sign this effect). We summarize this discussion as follows.

Proposition 1 *In the privilege contest model, the rich player exerts more effort than the poor player. Moreover, a common increase in wealth increases total effort. An isolated increase in the poor player's wealth always increases the equilibrium efforts of both players. An isolated increase in the rich player's wealth has a positive effect on his own effort but a negative effect on the effort of the poor player.*

We finally discuss the effects of wealth inequality. We observe that decreasing inequality in the sense of making the poor richer, or increasing inequality in the sense of making the rich richer, both may increase total effort. Therefore, there is no systematic relationship between wealth inequality and effort in the privilege contest model. We therefore study the effect of more wealth inequality when total wealth is constant. More precisely, we study the effect of a mean-preserving spread (MPS) in wealth. In the Appendix, we show that when m is "small enough" then a small MPS increases total effort. Under CARA this holds when $m < 2^{-\frac{1}{2}} \simeq .707$. When u is quadratic, the condition is $m < 1$. When u has CRRA ρ, this holds iff $\rho(\frac{1}{2} - m^2) > \frac{1}{2}$. In the numerical example of Figure 2, which has $\rho = m = 2$, aggregate effort in point D ($w_a = w_b = 20$) is 29.19. Redistributing one unit of wealth results in a lower aggregate effort (29.14).

3 The ability contest model

In the ability contest model, effort is non-monetary. Our principal interpretation is a situation in which agents exert physical or mental efforts that increase their abilities, and thus put them in a better position to win a contest. Competitive sports, but also education filters, are examples of such contests. In this model, player i's expected utility equals

$$U_i = \Pi_i u(w_i + r) + (1 - \Pi_i) u(w_i) - c(x_i),$$

with $c' > 0$ and $c'' \geq 0$. As before, we assume that $u(\cdot)$ is concave, which represents decreasing marginal utility of wealth (or risk aversion). The marginal utility of effort is

$$\frac{\partial U_i}{\partial x_i} = \frac{\partial \Pi_i}{\partial x_i} [u(w_i + r) - u(w_i)] - c'(x_i).$$

The key property in this contest model is that the marginal cost of exerting effort is independent of wealth. As argued earlier, if this marginal cost is strictly increasing we can relax the assumption that $m \leq 1$ for a Nash equilibrium in pure strategies to exist. As in the privilege contest, the marginal cost of effort does not depend on the effort of the opponent, while the marginal benefit does in the way described by (3).

Again, we consider first the simpler symmetric case, $w_a = w_b = w$. Using (2), the equilibrium condition for the symmetric equilibrium x is now given by

$$H_A(w) \stackrel{\text{def}}{=} \frac{m}{4x} [u(w + r) - u(w)] - c'(x) = 0.$$

Observe that wealth only affects the term $\frac{m}{4x}[u(w + r) - u(w)]$, and that $H'_A(w)$ is negative under $u'' < 0$. This effect represents the second effect mentioned in the Introduction that wealth reduces the marginal benefit of winning the contest. The rich is relatively less motivated to win the contest than the poor, and effort is *an inferior good* in this model.

In the ability contest model, the direction of the effects is thus reversed compared to the privilege contest model. If the rich agent gets wealthier, he will reduce his effort, which will lower the marginal benefit of the effort of the poorer opponent. So the latter reduces her effort as well and total effort falls.

On the other hand, if the poorer agent gets wealthier, she reduces her effort, which increases the marginal benefit of the richer agent who responds by adjusting his effort upwards. The effect on total effort may then be positive, especially when the wealth gap, and therefore the difference in effort that governs (3), is large (see the Appendix A.5 for a sufficient condition to sign this effect).

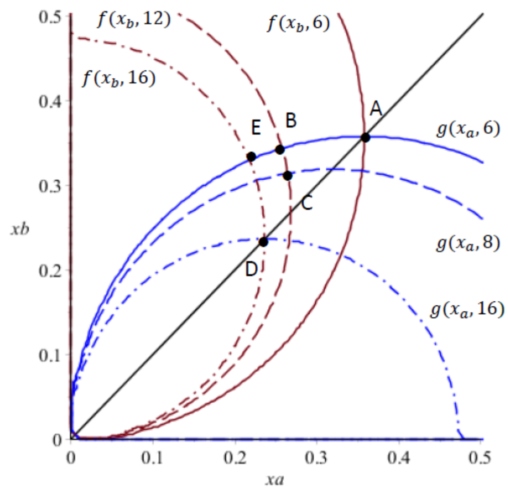

Figure 2. Equilibria in the ability contest model for different wealth combinations.

Figure 2 further illustrates these results. For this example $u(w) = \log(w)$ (i.e., CRRA $\rho = 1$), $c(x) = x^2$, $m = 2$, and $r = 4$. The best-response functions of players a and b are drawn for $w_a = 6, 12$ and 16, and for $w_b = 6, 8$, and 16. An increase in wealth decreases the best-response functions in the ability contest model. Thus, point A represents a symmetric equilibrium with low wealth ($w_a = w_b = 6$), and the move from A to D illustrates the effect of a common increase in wealth from 6 to 16. Similarly, point B represents an equilibrium with $w_a = 12 > w_b = 6$, and the move from B to E illustrates the effect of an increase in w_a, resulting in a downward adjustment in both effort levels. The move from B to C on the other hand, represents an increase in w_b from 6 to 8, resulting in opposing adjustments in the effort levels of the two players. This leads to the following results.

Proposition 2 *In the ability contest model, the rich player exerts less effort than the poor player. A common increase in wealth decreases total effort. An isolated increase in the rich player's wealth always reduces the equilibrium* *effort of both players. An isolated increase in the poor player's wealth reduces her own effort but increases the effort of the rich player.*

We now discuss the effects of wealth inequality. As in the privilege contest, we first observe that there is no systematic relationship between wealth inequality and effort in the ability contest model. Indeed, decreasing inequality in the sense of making the poor richer, or increasing inequality in the sense of making the rich richer, both may decrease total effort. We then examine the effect of a small MPS in wealth. In the Appendix, we show that when the contest decisiveness parameter m is sufficiently small compared to the difference in curvature of $u(\cdot)$ and $c(\cdot)$, then aggregate effort will rise following the introduction of a small wealth inequality. In the numerical example of Figure 2, when $w_a = w_b = 6$, aggregate effort is .7147 and a one unit wealth transfer from a to b lowers aggregate effort (.7133).

4 The rent-seeking contest model

In the rent-seeking contest model, both rent and effort are monetary. This model can accommodate many contest-type situations including lobbying, marketing, and litigation activities where both the rent and the effort have a direct monetary counterpart.[5] In this model, player i's expected utility equals

$$U_i = \Pi_i u(w_i + r - x_i) + (1 - \Pi_i)u(w_i - x_i). \tag{5}$$

The concavity of $u(\cdot)$ is usually interpreted as risk aversion (Cornes and Hartley 2012), and we retain this interpretation in what follows. The marginal utility of effort is now

$$\frac{\partial U_i}{\partial x_i} = \frac{\partial \Pi_i}{\partial x_i}[u(w_i+r-x_i)-u(w_i-x_i)]-[\Pi_i u'(w_i + r - x_i) + (1 - \Pi_i)u'(w_i - x_i)] .$$

Unlike in the privilege or ability contest model, the marginal cost of a player's effort, which corresponds to the expected marginal utility (the second square bracket term), increases when the opponent raises his or her effort.

[5]The economics literature on contests has traditionally (and often implicitly) assumed that both the rent and the effort are monetary. For instance, an important focus in this literature has concerned the rate of rent dissipation, i.e., $\frac{\Sigma_i x_i}{r}$, which assumes that the rent and the efforts are expressed in the same–typically monetary–unit.

Let $w_a = w_b = w$. Using (2), the equilibrium condition for the symmetric equilibrium x is now given by

$$H_{RS}(w) \stackrel{\text{def}}{=} \frac{m}{4x}[u(w+r-x) - u(w-x)] - \frac{1}{2}[u'(w+r-x) + u'(w-x) = 0.$$

Observe that wealth affects both terms of the equilibrium condition. We have

$$H'_{RS}(w) = \frac{m}{4x}[u'(w+r-x) - u'(w-x)] - \frac{1}{2}[u''(w+r-x) + u''(w-x)],$$

so that the first term of the *rhs* is negative while the second term is positive. This illustrates the two opposite effects of wealth on rent seeking efforts.

Let us now define two lotteries: a uniformly distributed lottery $\widetilde{z} =_d \mathrm{U}(w-x, w-x+r)$ and a binary lottery $\widetilde{y} =_d (w-x+r, \frac{1}{2}; w-x, \frac{1}{2})$. Then it is easy to check that $H'_{RS}(w) > 0$ is equivalent to

$$\frac{-Eu''(\widetilde{z})}{Eu'(\widetilde{z})} < \frac{-Eu''(\widetilde{y})}{Eu'(\widetilde{y})}.$$

In other words, a common increase in wealth in the rent-seeking contest model increases efforts depending on how absolute risk aversion is affected by a change in background risk from lottery $\widetilde{z}$ to lottery $\widetilde{y}$. It is then immediately obtained that when u displays CARA then $H'_{RS}(w) = 0$. It is also easily observed that $H'_{RS}(w) = 0$ under a quadratic utility.[6] Given that the binary lottery ($\widetilde{y}$) is a MPS of the uniform lottery ($\widetilde{z}$), the sign of $H'_{RS}(w)$ is positive (resp. negative) if the MPS of a background risk increases (resp. decreases) the coefficient of absolute risk aversion. Let us now introduce the following definition.

Definition 1 *Let Ω be the class of utility functions so that a MPS of a background risk increases absolute risk aversion.*

It sounds intuitive that additional background risk induces greater risk aversion, i.e., $u \in \Omega$. Eeckhoudt, Gollier and Schlesinger (1996, Prop. 3) show, however, that the conditions on $u(\cdot)$ so that extra background risk

[6] Observe that $E\widetilde{z} = E\widetilde{y} = w - x + \frac{1}{2}r$. If $u(w) = w - \frac{\beta}{2}w^2$, then $\frac{-Eu''(\widetilde{y})}{Eu'(\widetilde{y})} = \frac{-Eu''(\widetilde{z})}{Eu'(\widetilde{z})} = \frac{\beta}{1-\beta(w-x+\frac{1}{2}r)}$.

makes an agent more risk averse are complex, involving restrictions on higher-order attitudes towards risk, such as the degree of temperance, i.e., $-u''''/u'''$. A necessary condition for $u \in \Omega$ is that risk aversion increases when a zero-mean background risk is introduced. Gollier and Pratt (1996) called this condition "risk vulnerability" and it is a stronger condition than the familiar DARA (decreasing absolute risk aversion). The intuition for this result may be given as follows. Investing in a contest is like gambling, where one spends money to increase the probability of winning the monetary prize. For the same reason that gambling activities should be reduced under increased risk aversion, efforts in a contest should also be reduced with increasing risk aversion (Treich 2010). By a similar reasoning, an increase in wealth —which reduces risk aversion under DARA— should increase effort in a contest. Thus $u \in \Omega$ (implying DARA) ensures that effort in the rent-seeking contest model is *a normal good.*

The first round effects of a wealth increase can then be presented as follows. If the rich agent gets wealthier, he will increase effort which not only lowers the return to the poor agent's effort (through (3)) but also raises the marginal cost of that effort. Hence the poor agent reduces her effort. If the poor agent gets wealthier, she increases her effort which raises the marginal return of the rich agent's effort but also raises the marginal cost of that effort. The first effect, however, will be of second order when the two agents are about equally wealthy, in which case we expect to see a reduction in the rich agent's effort. The opposite is true if there is a substantial wealth inequality; in that case, total effort will increase.

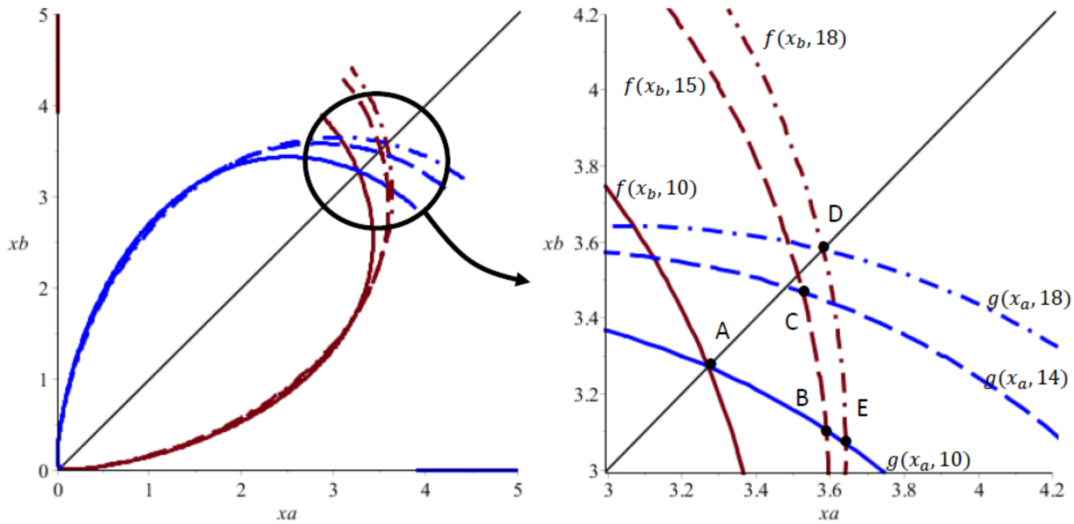

Figure 3. Equilibria in the rent-seeking model for different wealth combinations.

Figure 3 illustrates the different wealth effects occurring under $u \in \Omega$. It is assumed that $u(w) = \log(w)$ (i.e., CRRA ρ=1), $m = \frac{3}{2}$, $r = 10$. The best-response functions of players a and b are drawn for $w_a = 10, 15$ and 18, and for $w_b = 10, 14$ and 18. Observe that unlike in the privilege or ability contest the best-response functions cross the 45^0-line with a strictly negative and finite slope. Observe also that the best-response functions again display a discontinuity but without upsetting the existence of a Nash equilibrium in pure strategies for our range of parameters. Point A is a symmetric equilibrium where $w_a = w_b = 10$. A common increase in wealth by 8 units moves the equilibrium to D. Indeed, wealth increases the equilibrium effort. Point B is an asymmetric equilibrium with $w_a = 15 > w_b = 10$. The move from B to E is then because of an increase in w_a from 15 to 18: x_a increases, but x_b falls. Conversely, the move from B to C is because of an increase in w_b from 10 to 14. While raising x_b, this leads to a fall in x_a, illustrating the dominance of the increased marginal cost of a's effort. The same is true for the move from E to D. The results are demonstrated in Appendix A.6 and summarized below.

Proposition 3 *Suppose that $u \in \Omega$. In a rent-seeking contest model, the rich player exerts more effort than the poor player. An isolated increase in the wealth of the rich player increases that player's effort, but reduces the poor player's effort. An isolated increase in the wealth of the poor player increases that player's effort.*

We finally discuss the effect of wealth inequality on aggregate effort in the rent-seeking contest model. For the reason discussed earlier, there is no effect of wealth distribution across players under CARA or quadratic utility. In the Appendix, we prove under CRRA preferences and when the stake of the contest, $\frac{r}{w}$, is small, that a small MPS in wealth reduces aggregate effort. In the numerical example of Figure 3, which has CRRA $\rho = 1$, aggregate effort when $w_a = w_b = 18$ is 7.159. Redistributing one unit of wealth leads to an aggregate effort of 7.155. Thus even with a $\frac{r}{w} = .55$, the prediction holds. We also provide a numerical example suggesting that $u''' < 0$ ("imprudence") may be a necessary condition for a MPS in wealth to raise aggregate effort.

5 Summary and conclusion

The archetype of a contest is warfare. There is an old Latin expression (often attributed to Cicero), "pecunia nervus belli", meaning that "money is the sinews of war". That is, wealth is expected to play a central role in warfare. But is this theoretically true? We base our answer on a theoretical analysis of strategic models of contests. These models capture economic situations with elements of warfare. The simplest conclusion we can offer from our analysis is that the relationship between contestants' efforts and wealth is strongly "contest-dependent". Besides, for each contest the results often depend on functional forms and parameter values. Hence, our analysis does not support without qualifications general claims that the rich lobby more, nor that low economic growth and inequality increase conflict.

The more precise answer is that wealth effects critically depend on the nature of the rent and of the type of efforts exerted in a contest. They depend in particular on whether the rent and/or efforts are commensurable with wealth within the utility function. We have especially stressed the importance of the property of decreasing marginal utility of wealth. This property plays a fundamental role in our analysis through two basic effects. First, wealth decreases the marginal cost of monetary effort. Second, wealth decreases the marginal benefit of winning a monetary rent. The first effect tends to increase efforts, as we have shown in the privilege contest model. The second effect tends to decrease efforts, as we have shown in the ability contest model. Therefore, the disparate effects of wealth in contests that we colloquially discussed in the Introduction may well reflect these two fundamentally opposing forces that our models identify.

However, these basic effects go in an opposite direction when both the rent and efforts are monetary. Therefore the total effect of wealth is complex and potentially limited, as we have shown in our rent-seeking contest model. In the special, but common, case of a CARA utility function, the two effects exactly offset each other with wealth having no impact on effort. Moreover, we have shown that wealth increases effort in the rent-seeking contest model under the assumption on the utility function that more background risk increases risk aversion in a single decision making setting. This assumption involves higher-order derivatives of the utility function, and is stronger than DARA. All the theoretical results of the three strategic contest games are derived in the Appendix and are summarized in Table 1.

Table 1. Wealth effects in the privilege, ability, and rent-seeking contest models under $w_a \geq w_b$.[a]

Contest models:		Privilege	Ability	Rent-seeking
Rich vs poor	$x_a - x_b$	+	−	+
Isolated increase in w_a	$\frac{\partial x_a}{\partial w_a}$	+	−	+
	$\frac{\partial x_b}{\partial w_a}$	−	−	−
	$\frac{\partial (x_a+x_b)}{\partial w_a}$	+ iff (A.13)	−	+ iff (A.22)
Isolated increase in w_b	$\frac{\partial x_a}{\partial w_b}$	+	+	?
	$\frac{\partial x_b}{\partial w_b}$	+	−	+
	$\frac{\partial (x_a+x_b)}{\partial w_b}$	+	− iff (A.18)	+ iff (A.23)
Common increase	$\frac{\partial x_i}{\partial w_i}\Big\vert_{\mathrm{d}w_a=\mathrm{d}w_b}^{w_a=w_b}$	+	−	+
	$\frac{\partial (x_a+x_b)}{\partial w_i}\Big\vert_{\mathrm{d}w_a=\mathrm{d}w_b}^{w_a=w_b}$	+	−	+
"Small" MPS in wealth	$\frac{\partial x_i}{\partial w_i}\Big\vert_{\mathrm{d}w_a=-\mathrm{d}w_b}^{w_a=w_b}$	+ iff (A.16)	+ iff (A.19)	+ iff (A.24)

[a]Symbols + and − indicate the sign of the effects mathematically described in the second column. Symbol ? indicates that this sign is indeterminate. In the rent-seeking contest model, we assume $u \in \Omega$ (cf. Definition 1).

Table 1 also illustrates the subtle effects of isolated changes in wealth compared to other possible parametric changes. Nti (1999) for instance examines in an asymmetric contest the effect of changes in the valuation of the rent, and concludes that "competition becomes keener when the gap between the contestants is not too large" (Nti 1999, p. 425). Based on this insight, one would expect that increasing the wealth of the rich should decrease aggregate efforts, while increasing the wealth of the poor should increase aggregate efforts. Table 1 illustrates a much more complex pattern. Indeed, the total effect of an isolated increase in wealth of either the poor or the rich may be always positive, or always negative. Moreover this effect critically depends on the type of contest (and the curvature of the utility function of wealth).

The last observation relates to our analysis of the effects of wealth inequality. These effects are the most complex. To illustrate, take the privilege contest model. In this model, an isolated increase in wealth may always increase aggregate effort. Therefore, a decrease in inequality (through an increase in the wealth of the poor) or an increase in inequality (through an increase in the wealth of the rich) both increase aggregate effort. Observe then that wealth redistribution, i.e., transferring money from the rich to the poor, combines the first effect and the opposite of the second effect (for a fixed total wealth). It thus essentially involves two opposing effects, and it becomes very difficult to sign the effect of such a MPS in wealth without further restricting the functional forms. In fact, we have shown in our three contest models that the effect of a MPS in wealth depends on the property of the CSF, as well as on the sign of the higher-order derivatives of the utility functions. Interestingly, these results stand in sharp contrast with the "neutrality result" concerning the effect of wealth redistribution in the celebrated private provision of public goods model (Bergstrom, Blume and Varian 1986). A simple implication of our analysis is that the consequences of a wealth redistribution policy in terms of political stability and social peace are by no means obvious in general.

To conclude, let us mention some natural extensions to our results. To start with, one may wish to consider other CSFs, an arbitrary number of players and other dimensions of heterogeneity (e.g., on the cost or value of rent). We may also want to assume that the rent itself depends on wealth. As an illustration of this assumption, observe that a successful lobby activity of a large firm is likely to give more benefits than that of a small firm. Intuitively, this effect increases the marginal benefit of winning the rent, and should thus in general reinforce the first positive effect of wealth on efforts. Interestingly though, this effect holds even with a linear utility function of wealth, and is present in Hirshleifer (1991) and Nti (1999). One might also want to explore welfare effects. This could be interesting in that an increase in the wealth of one or more players need not have a positive effect on overall welfare despite increasing utility. This is because of the strategic effects that may serve to increase overall effort. However, a general study of welfare effects in contest models must also explicitly discuss to which extent efforts are socially (un)productive. Finally, it could also be interesting to explore dynamic effects: wealth affects conflict, which in turn affects wealth, and so on. Such a dynamic analysis would permit a better understanding of the somewhat elusive relationship between power and money.

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A Appendix

A.1 General comparative statics method

In our analysis, we study the effects of several types of wealth changes. We present here a method and some results about the conditions that determine the sign of these wealth effects in a general class of strategic models. This class includes the three contest models considered in the paper.[7] Theorem 1 below provides a simple single crossing property that will turn out to be instrumental throughout the paper, while Theorem 2 derives a condition for signing the effect of a MPS in wealth.

We consider a strategic game with two players, $i = a, b$, in which the only source of heterogeneity is wealth w_i. We assume without loss of generality that a is more wealthy than b: $w_a \geq w_b$ (with $w_a = w_b$ corresponding to the symmetric situation). Each player i chooses an effort level x_i from a convex feasible set. Remember that the best-response functions are denoted $f(x_b, w_a)$ and $g(x_a, w_b)$ and that (x_a, x_b) constitute a Nash equilibrium for the game with initial wealth (w_a, w_b) when

$$x_a = f(g(x_a, w_b), w_a), \tag{A.1}$$

$$x_b = g(f(x_b, w_a), w_b). \tag{A.2}$$

We write $x_a(w_a, w_b)$ and $x_b(w_a, w_b)$ as the unique pair of equilibrium effort levels for this game. We now introduce the following single-crossing property.

Theorem 1 *Suppose that* $x_a = x_b \Longrightarrow \frac{\partial x_a(w_a,w_b)}{\partial w_a} > (<) \frac{\partial x_b(w_a,w_b)}{\partial w_a}$. *Then* $w_a > w_b \Longrightarrow x_a(w_a, w_b) > (<) x_b(w_a, w_b)$.

Proof of Theorem 1

We need to prove that if (i) $x_a = x_b \Longrightarrow \frac{\partial x_a(w_a,w_b)}{\partial w_a} > \frac{\partial x_b(w_a,w_b)}{\partial w_a}$, then (ii) $w_a > w_b \Longrightarrow x_a(w_a, w_b) > x_b(w_a, w_b)$. Since $x_a(w_b, w_b) = x_b(w_b, w_b)$ (i.e., the unique equilibrium is the symmetric equilibrium), it follows from (i) that $\frac{\partial x_a(w_a,w_b)}{\partial w_a}|_{w_a=w_b} > \frac{\partial x_b(w_a,w_b)}{\partial w_a}|_{w_a=w_b}$. If for some $w_a > w_b$, we have $x_a(w_a, w_b) \leq$

[7] Our strategic contest models belong to the class of "aggregative games" for which each individuals' payoffs only depend on their own effort and on the aggregate efforts of all players. It has been shown that aggregative games display special features that make their analysis simpler under some conditions (Bergstrom and Varian 1985, Corchon 1994, Acemoglu and Jensen 2013). Nevertheless, the following preliminary results are fairly general, and not restricted to aggregative games.

$x_b(w_a, w_b)$ this implies, due to the continuity of best responses, that there exists $w_c \in (w_a, w_b]$, such that $x_a(w_c, w_b) = x_b(w_c, w_b)$ and $\frac{\partial x_a(w_a,w_b)}{\partial w_a}|_{w_a=w_c} \leq \frac{\partial x_b(w_a,w_b)}{\partial w_a}|_{w_a=w_c}$, contradicting (i). Hence, we must have (ii) $x_a(w_a, w_b) > x_b(w_a, w_b)$. The case with reverse inequalities can be proved in an analogous fashion. This proves Theorem 1.

This theorem implies that when $\frac{\partial x_a(w_a,w_b)}{\partial w_a}|_{w_a=w_b} > \frac{\partial x_b(w_a,w_b)}{\partial w_a}|_{w_a=w_b}$, player a exerts more effort than player b (when $w_a \geq w_b$). Thus, to compare within an equilibrium the relative effort of the rich and poor player, it is sufficient to examine at the symmetric equilibrium how each player comparatively reacts to an increase in the wealth of player a. This result is illustrated in Figure A1.

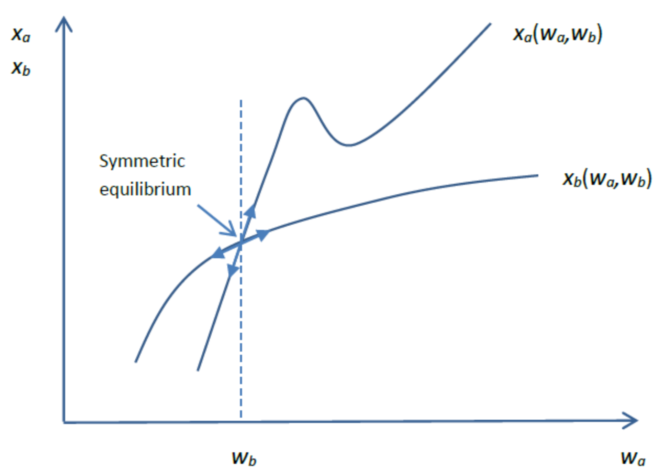

Figure A1. Single crossing of the equilibrium efforts.

We have characterized a property of the equilibrium in an asymmetric game. In addition, we will assume in the following that the condition $1 - f_1 g_1 > 0$ is always satisfied. Note that this is the case if we assume that the equilibrium is locally stable, or $|f_1 g_1| < 1$ (see, e.g., Mas-Colell, Whinston and Green 1995, p. 414). We discuss this condition in A.3 below for our three contest models.

Implicit differentiation of (A.1)-(A.2) gives the effects of isolated increases in wealth:

$$\frac{\partial x_a}{\partial w_a} = \frac{f_2}{1 - f_1 g_1}, \tag{A.3}$$

$$\frac{\partial x_b}{\partial w_a} = \frac{g_1 f_2}{1 - f_1 g_1}, \tag{A.4}$$

$$\frac{\partial x_a}{\partial w_b} = \frac{f_1 g_2}{1 - f_1 g_1}, \tag{A.5}$$

$$\frac{\partial x_b}{\partial w_b} = \frac{g_2}{1 - f_1 g_1}, \tag{A.6}$$

where the numerical subscripts with f and g denote partial derivatives and these functions are all evaluated at equilibrium. Thus, an increase in w_a increases player a's effort iff $f_2 > 0$ and increases player's b effort iff $g_1 f_2 > 0$. The corresponding effects on aggregate effort are

$$\frac{\partial x_a}{\partial w_a} + \frac{\partial x_b}{\partial w_a} = \frac{f_2(1+g_1)}{1 - f_1 g_1}, \text{ and } \frac{\partial x_a}{\partial w_b} + \frac{\partial x_b}{\partial w_b} = \frac{g_2(1+f_1)}{1 - f_1 g_1}. \tag{A.7}$$

In a symmetric equilibrium (SE), $f_i = g_i$ $(i = 1, 2)$. In that case, the change in individual effort following a common wealth increase is

$$\frac{\partial x_i}{\partial w_i}\Big|^{\mathrm{SE}}_{\mathrm{d}w_a = \mathrm{d}w_b} = \frac{f_2}{1 - f_1}. \tag{A.8}$$

Finally, when wealth is redistributed from b to a, $\mathrm{d}w_a = -\mathrm{d}w_b$. Then

$$\frac{\mathrm{d}x_a}{\mathrm{d}w_a}\Big|_{\mathrm{d}w_a = -\mathrm{d}w_b} = \frac{f_2 - f_1 g_2}{1 - f_1 g_1}, \text{ and } \frac{\mathrm{d}x_b}{\mathrm{d}w_a}\Big|_{\mathrm{d}w_a = -\mathrm{d}w_b} = \frac{g_1 f_2 - g_2}{1 - f_1 g_1}.$$

In a symmetric equilibrium, a wealth transfer from b to a has no first-order effect on aggregate effort since

$$\frac{\mathrm{d}x_a}{\mathrm{d}w_a}\Big|^{\mathrm{SE}}_{\mathrm{d}w_a = -\mathrm{d}w_b} = -\frac{\mathrm{d}x_b}{\mathrm{d}w_a}\Big|^{\mathrm{SE}}_{\mathrm{d}w_a = -\mathrm{d}w_b} = \frac{f_2}{1 + f_1}.$$

The second-order effect of such an MPS in wealth is given by the following theorem.

Theorem 2 *Consider a symmetric equilibrium. Let the stability condition $f_1^2 < 1$ be satisfied. The second-order effect of a MPS in wealth $dw_a = -dw_b$ on aggregate effort $x_a + x_b$ is given by*

$$\frac{(f_2)^2 f_{11} - 2(1+f_1) f_2 f_{12} + (1+f_1)^2 f_{22}}{(1+f_1)(1-f_1^2)}. \tag{A.9}$$

The numerator is a quadratic form in the Hessian of $f(\cdot)$.[8] *The denominator is positive under the stability condition.*

Proof of Theorem 2

For a symmetric equilibrium,

$$x_a = f(f(x_a, w_b), w_a) \text{ and } x_b = f(f(x_b, w_a), w_b).$$

These expressions may be solved for the reduced form expressions for equilibrium effort:

$$x_a = F(w_a, w_b) \text{ and } x_b = F(w_b, w_a).$$

Theorem 2 is now proven with the help of two lemmas.

Lemma 1 *A small redistribution in wealth $dw_a = -dw_b = t$ increases aggregate effort $x_a + x_b$ iff $F_{11} - 2F_{12} + F_{22}$ evaluated at (w, w) is positive.*

Proof of Lemma 1

Starting from an equal wealth distribution (w, w), the new effort level for player a following a transfer t from b to a is then

$$x_a(w+t, w-t) = F(w+t, w-t) \simeq F(w, w) + (F_1 - F_2)t + \frac{1}{2}(F_{11} - 2F_{12} + F_{22})t^2,$$

where F_i means the partial w.r.t. the ith argument and all derivatives are evaluated at (w, w). Likewise, the new effort level for player b is approximately

$$x_b(w+t, w-t) = F(w-t, w+t) \simeq F(w, w) - (F_1 - F_2)t + \frac{1}{2}(F_{11} - 2F_{12} + F_{22})t^2.$$

Hence, aggregate equilibrium efforts are equal to

$$x_a(w + t, w - t) + x_b(w + t, w - t) \simeq 2F(w, w) + (F_{11} - 2F_{12} + F_{22})t^2.$$

Lemma 2 *At a symmetric equilibrium,*

$$F_{11} - 2F_{12} + F_{22} = \frac{(f_2)^2 f_{11} - 2(1 + f_1)f_2 f_{12} + (1 + f_1)^2 f_{22}}{(1 + f_1)(1 - f_1^2)},$$

where f_i (f_{ij}) denotes the first- (second-)order partial w.r.t. arguments i (ij).

---

[8] It can be written as $(-f_2, 1 + f_1)\begin{pmatrix} f_{11} & f_{12} \\ f_{12} & f_{22} \end{pmatrix}\begin{pmatrix} -f_2 \\ 1+f_1 \end{pmatrix}$. Moreover, it can be easily checked that this form equals zero under the conditions identified in the theorem in Bergstrom and Varian (1985, p. 717). These conditions ensure that the distribution of agent characteristics has no effect on aggregate effort.

Proof of Lemma 2

Let $f(x_b, w_a)$ be the best-response function for agent a, and $g(x_a, w_b)$ be the best-response function for agent b. Then

$$x_a = f(g(x_a, w_b), w_a).$$

Implicit differentiation then gives

$$\begin{aligned}
\mathrm{d}x_a &= f_1 g_1 \mathrm{d}x_a + f_2 \mathrm{d}w_a + f_1 g_2 \mathrm{d}w_b \\
F_1 &= \frac{\partial x_a}{\partial w_a} = \frac{f_2(g(x_a, w_b), w_a)}{1 - f_1(g(x_a, w_b), w_a) g_1(x_a, w_b)} \\
F_2 &= \frac{\partial x_a}{\partial w_b} = \frac{f_1(g(x_a, w_b), w_a) g_2(x_a, w_b)}{1 - f_1(g(x_a, w_b), w_a) g_1(x_a, w_b)}.
\end{aligned}$$

Differentiating one more time gives

$$\begin{aligned}
F_{11} = &\frac{1}{1 - f_1 g_1} \left\{ f_{21} g_1 \frac{\partial x_a}{\partial w_a} + f_{22} \right. \\
&\left. + \frac{f_2}{1 - f_1 g_1} \left[\left(f_{11} g_1 \frac{\partial x_a}{\partial w_a} + f_{12} \right) g_1 + f_1 g_{11} \frac{\partial x_a}{\partial w_a} \right] \right\} \\
= &\frac{1}{1 - f_1 g_1} \left\{ f_{21} g_1 \frac{f_2}{1 - f_1 g_1} + f_{22} \right. \\
&\left. + \frac{f_2}{1 - f_1 g_1} \left[\left(f_{11} g_1 \frac{f_2}{1 - f_1 g_1} + f_{12} \right) g_1 + f_1 g_{11} \frac{f_2}{1 - f_1 g_1} \right] \right\}
\end{aligned}$$

$$\begin{aligned}
F_{12} = &\frac{1}{1 - f_1 g_1} \left\{ f_{21} \left(g_1 \frac{\partial x_a}{\partial w_b} + g_2 \right) \right. \\
&\left. + \frac{f_2}{1 - f_1 g_1} \left[f_{11} \left(g_1 \frac{\partial x_a}{\partial w_b} + g_2 \right) g_1 + f_1 \left(g_{11} \frac{\partial x_a}{\partial w_b} + g_{12} \right) \right] \right\} \\
= &\frac{1}{1 - f_1 g_1} \left\{ f_{21} \left(g_1 \frac{f_1 g_2}{1 - f_1 g_1} + g_2 \right) \right. \\
&\left. + \frac{f_2}{1 - f_1 g_1} \left[f_{11} \left(g_1 \frac{f_1 g_2}{1 - f_1 g_1} + g_2 \right) g_1 + f_1 \left(g_{11} \frac{f_1 g_2}{1 - f_1 g_1} + g_{12} \right) \right] \right\}
\end{aligned}$$

$$
\begin{aligned}
F_{22} = & \frac{1}{1-f_1g_1}\left\{f_{11}\left(g_1\frac{\partial x_a}{\partial w_b}+g_2\right)g_2+f_1\left(g_{21}\frac{\partial x_a}{\partial w_b}+g_{22}\right)\right. \\
& \left.+\frac{f_1g_2}{1-f_1g_1}\left[f_{11}\left(g_1\frac{\partial x_a}{\partial w_b}+g_2\right)g_1+f_1\left(g_{11}\frac{\partial x_a}{\partial w_b}+g_{12}\right)\right]\right\} \\
= & \frac{1}{1-f_1g_1}\left\{f_{11}\left(g_1\frac{f_1g_2}{1-f_1g_1}+g_2\right)g_2+f_1\left(g_{21}\frac{f_1g_2}{1-f_1g_1}+g_{22}\right)\right. \\
& \left.+\frac{f_1g_2}{1-f_1g_1}\left[f_{11}\left(g_1\frac{f_1g_2}{1-f_1g_1}+g_2\right)g_1+f_1\left(g_{11}\frac{f_1g_2}{1-f_1g_1}+g_{12}\right)\right]\right\}.
\end{aligned}
$$

At a symmetric equilibrium, $f_{12} = f_{21} = g_{21} = g_{12}$, $f_1 = g_1$, $f_2 = g_2$, and $f_{22} = g_{22}$. Then, using the above expressions, it can be shown that

$$
F_{11} - 2F_{12} + F_{22} = \frac{(f_2)^2 f_{11} - 2(1+f_1)f_2 f_{12} + (1+f_1)^2 f_{22}}{(1+f_1)(1-f_1^2)}.
$$

The numerator is a quadratic form in the Hessian of the best-response function $f(x_2, w_1)$:

$$
\begin{bmatrix} -f_2 & 1+f_1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} \\ f_{12} & f_{22} \end{bmatrix} \begin{bmatrix} -f_2 \\ 1+f_1 \end{bmatrix}.
$$

The denominator will be positive under the stability assumption: $|f_1g_1| = |f_1^2| = f_1^2 < 1 \Longrightarrow |f_1| < 1$.

The proof of Theorem 2 then follows immediately from Lemmas 1 and 2.

A.2 Power-logistic contest success functions

In this section, we display the properties of the power-logistic CSF which are useful to derive our results below in each strategic contest model. Remember that the CSF for player a is given by $\Pi_a(x_a, x_b) = p(x_a, x_b) = \frac{x_a^m}{x_a^m + x_b^m}$,where $m > 0$. Then $\Pi_b(x_a, x_b) = 1 - p(x_a, x_b) = \frac{x_b^m}{x_a^m + x_b^m}$. The derivatives of $p(x_a, x_b)$

are (where $\overset{\text{SE}}{=}$ denotes evaluation at $x_a = x_b = x$)

$$
\begin{aligned}
p_1 &= \frac{m}{x_a}\Pi_a\Pi_b \overset{\text{SE}}{=} \frac{1}{4}\frac{m}{x} > 0 \text{ and } p_2 = -\frac{m}{x_b}\Pi_a\Pi_b \overset{\text{SE}}{=} -\frac{1}{4}\frac{m}{x} < 0;\\
p_{12} &= \frac{m^2}{x_a x_b}\Pi_a\Pi_b(\Pi_a - \Pi_b) > 0 \text{ iff } x_a > x_b, \text{ and } \overset{\text{SE}}{=} 0;\\
p_{11} &= \frac{m}{x_a^2}\Pi_a\Pi_b[-1 + m(\Pi_b - \Pi_a)] \overset{\text{SE}}{=} -\frac{1}{4}\frac{m}{x^2};\\
p_{22} &= \frac{m}{x_b^2}\Pi_a\Pi_b[1 - m(\Pi_a - \Pi_b)] \overset{\text{SE}}{=} \frac{1}{4}\frac{m}{x^2};\\
p_{111} &= 2\frac{m}{x_a^3}\Pi_a\Pi_b - \frac{m^2}{x_a^3}\Pi_a\Pi_b^2 + \frac{m^2}{x_a^3}\Pi_a^2\Pi_b - \frac{m^2}{x_a^3}\Pi_a\Pi_b(\Pi_b - \Pi_a)\\
&\quad + \frac{m^2}{x_a^3}\Pi_a\Pi_b[-1 + m(\Pi_a - \Pi_b)](\Pi_b - \Pi_a) - 2\frac{m^3}{x_a^3}\Pi_a^2\Pi_b^2\\
&\overset{\text{SE}}{=} \frac{1}{2}\frac{m}{x^3} - \frac{1}{8}\frac{m^3}{x^3};\\
p_{122} &= \frac{m}{x_a}p_{22}(\Pi_b - \Pi_a) - 2\frac{m^3}{x_a x_b^2}\Pi_a^2\Pi_b^2 \overset{\text{SE}}{=} -\frac{1}{8}\frac{m^3}{x^3};\\
p_{112} &= \frac{m}{x_a}\left(p_{12} - p_2\frac{1}{x_b}\right)(\Pi_b - \Pi_a) + 2\frac{m^3}{x_a^2 x_b}\Pi_a^2\Pi_b^2 \overset{\text{SE}}{=} \frac{1}{8}\frac{m^3}{x^3}.
\end{aligned}
$$

For future reference, we also note that

$$3p_{112} - p_{111} \overset{\text{SE}}{=} \frac{1}{2}\frac{m}{x^3}(m^2 - 1).$$

A.3 Existence, uniqueness and stability

This section builds on the literature on strategic contests to identify conditions ensuring the existence of a unique pure Nash equilibrium. Remember that the payoff function in the privilege, ability, and rent-seeking contest models can be written respectively:

$$
\begin{aligned}
U_i &= u(w_i - x_i) + \Pi_i r.\\
U_i &= \Pi_i u(w_i + r) + (1 - \Pi_i)u(w_i) - c(x_i),\\
U_i &= \Pi_i u(w_i + r - x_i) + (1 - \Pi_i)u(w_i - x_i).
\end{aligned}
$$

Observe first that a sufficient condition for these payoff functions to be concave in x_i is that Π_i is concave in x_i, which under (2) implies $m \leq 1$. This assumption is enough to ensure that best response functions are unique and continuous in each contest model. We now derive a sufficient condition for existence and uniqueness in each contest model.

Proposition 4 *There exists a unique equilibrium:*
i) in the privilege contest model, if $\frac{-xu''(w-x)}{u'(w-x)} > m-1$;
ii) in the ability contest model, if $\frac{xc''(x)}{c'(x)} > m-1$;
iii) in the rent-seeking contest model, if $u(\cdot)$ *has non-increasing absolute risk aversion and* $m \leq 1$.

Proof of Proposition 4
We follow the proof of Szidarovszky and Okuguchi (1997). They show that there always exists a unique equilibrium when the form of the payoff function for each player i can be written as follows:

$$U_i = \frac{y_i}{\sum_j y_j} - g_i(y_i) \text{ with } g_i' > 0 \text{ and } g_i^{''} > 0.$$

In the privilege contest model, we obtain this form of the payoff function under the following change in variable $g_i(y_i) = -u(w_i - y_i^{\frac{1}{m}})/r$. Then it is immediate that $g_i' > 0$, and that $g_i'' > 0$ iff $\frac{-xu''(w-x)}{u'(w-x)} > m-1$. In the ability contest model, we obtain the above form of the payoff function under the following change in variable $g_i(y_i) = c(y_i^{\frac{1}{m}}/(u(w_i + r) - u(w_i)))$. Then it is immediate that $g_i' > 0$, and that $g_i'' > 0$ iff $\frac{xc''(x)}{c'(x)} > m-1$. Therefore, under $u'' < 0$ and $c'' > 0$, conditions i) and ii) hold as soon as $m < 1$. Finally, Yamazaki (2009) proves that there always exists a unique equilibrium in the rent-seeking contest model with CSF $\frac{\Phi(x_a)}{\Phi(x_a)+\Phi(x_b)}$ under non-increasing absolute risk aversion and $\Phi(\cdot)$ concave, which for the power-logistic function corresponds to $m \leq 1$. This proves Proposition 4.

We now discuss the condition $1 - f_1 g_1 > 0$. Throughout our analysis, we assume that this condition is satisfied. We first show that this is always the case in the privilege and ability contest models, and then resort to a stability condition which ensures that it is the case as well in the rent-seeking contest model. In the privilege contest model, f_1 and g_1 have opposite signs; see Appendix A.4. This implies $f_1 g_1 < 0$ and the condition is satisfied. Similarly, in the ability contest model, f_1 and g_1 also have opposite signs; see Appendix A.5. Therefore $f_1 g_1 < 0$ is also satisfied in that model. In the rent-seeking contest model, f_1 and g_1 need not have opposite signs, and the condition $1 - f_1 g_1 > 0$ is thus not necessarily verified. We thus impose a stability condition, i.e., $|f_1 g_1| < 1$, which ensures that the condition $1 - f_1 g_1 > 0$ is indeed satisfied. See Nti (1997) for a discussion of a related stability condition and of similar assumptions made in the literature on strategic contest models.

A.4 Privilege contest model

In this section, we demonstrate the results stated in Proposition 1 as well as some results supporting various statements regarding the effect of a MPS in wealth at the end of Section 2.

Let $U_i = u(w_i - x_i) + \Pi_i r$. The marginal willingness to pay for the privilege in terms of consumption, $\frac{\Pi_i}{u_i'}$, is decreasing (along the indifference curve) in consumption.[9] It also means that the marginal disutility of effort is increasing. Furthermore, we can express the dependency of this willingness to pay on wealth in terms of the coefficients of absolute risk aversion, $A_i \stackrel{\text{def}}{=} -\frac{u''(w_i - x_i)}{u'(w_i - x_i)}$, and absolute prudence, $P_i \stackrel{\text{def}}{=} -\frac{u'''(w_i - x_i)}{u''(w_i - x_i)}$:

$$\frac{\partial(-\frac{\mathrm{d}w_i}{\mathrm{d}r}|_{\mathrm{d}U_i=0})}{\partial w_i} = \frac{\Pi_i}{u_i'} A_i, \text{ and } \frac{\partial^2(-\frac{\mathrm{d}w_i}{\mathrm{d}r}|_{\mathrm{d}U_i=0})}{\partial w_i^2} = \frac{\Pi_i}{u_i'} A_i (2A_i - P_i). \tag{A.10}$$

Player a's best-response $f(x_b, w_a)$ is defined by the necessary first- and second-order conditions

$$\begin{aligned} -u'(w_a - f(x_b, w_a)) + p_1(f(x_b, w_a), x_b) r &= 0, \\ u''(w_a - f(x_b, w_a)) + p_{11}(f(x_b, w_a), x_b) r &< 0. \end{aligned}$$

The latter condition can then also be written as

$$\frac{p_{11}}{p_1} x_a = m \frac{1 - (\frac{x_a}{x_b})^m}{1 + (\frac{x_a}{x_b})^m} - 1 < A_a x_a. \tag{A.11}$$

Simple comparative statics show that

$$\begin{aligned} f_1 &= -\frac{p_{12}(x_a, x_b) r}{u''(w_a - x_a) + p_{11}(x_a, x_b) r} \text{ and} \\ f_2 &= \frac{u''(w_a - x_a)}{u''(w_a - x_a) + p_{11}(x_a, x_b) r} > 0, \end{aligned}$$

[9] As $-\frac{\mathrm{d}w_i}{\mathrm{d}r}|_{\mathrm{d}U_i=0} = \frac{\Pi_i}{u_i'}$, $\frac{\partial}{\partial r_i}(-\frac{\mathrm{d}w_i}{\mathrm{d}r}|_{\mathrm{d}U_i=0})|_{\mathrm{d}U_i=0} = \frac{\Pi_i^2}{u_i'} \frac{u_i''}{u_i'} < 0$.

where the inequality follows from the concavity of u and the second-order condition. Therefore player a's best-response increases when that player's wealth increases. Likewise, player b's best-response $g(x_a, w_b)$ satisfies the necessary first- and second-order conditions

$$\begin{aligned} -u'(w_b - g(x_a, w_b)) - p_2(x_a, g(x_a, w_b))r &= 0, \\ u''(w_b - g(x_a, w_b)) - p_{22}(x_a, g(x_a, w_b))r &< 0, \end{aligned}$$

where the last condition is equivalent to

$$\frac{p_{22}}{p_2} x_b = m \frac{1 - (\frac{x_b}{x_a})^m}{1 + (\frac{x_b}{x_a})^m} - 1 < A_b x_b. \tag{A.12}$$

Differentiating with respect to x_a and w_b, we obtain

$$\begin{aligned} g_1 &= \frac{-p_{12}(x_a, x_b)r}{-u''(w_b - x_b) + p_{22}(x_a, x_b)r}, \\ g_2 &= \frac{-u''(w_b - x_b)}{-u''(w_b - x_b) + p_{22}(x_a, x_b)r} > 0. \end{aligned}$$

At a symmetric equilibrium, $p_{12} = 0$ (cf. (3)) and therefore $f_1 = g_1 = 0$. Hence, at a symmetric equilibrium

$$\frac{\partial x_a}{\partial w_a} = f_2 > 0 \text{ and } \frac{\partial x_b}{\partial w_a} = 0,$$

and relying on Theorem 1 we can conclude that $x_a > x_b$ iff $w_a > w_b$. In view of (3), we can also conclude that $p_{12} > 0$. As a result, an isolated increase in the wealth of the poor player, b, increases both the equilibrium effort of that player (cf. (A.6) and $g_2 > 0$) as well as that of the rich player (cf. (A.5) and $f_1, g_2 > 0$). Hence, total equilibrium efforts also increase. Alternatively, an increase in the wealth of the rich player, a, increases that player's own equilibrium effort (cf. (A.3) and $f_2 > 0$) but reduces that of the poor player, b (cf. (A.4) and $g_1 < 0 < f_2$). We know from (A.7) that this total effect depends on $(1 + g_1)$. Observe now that $1 + g_1 > 0$ iff

$$[p_{12}(x_a, x_b) - p_{22}(x_a, x_b)]\, r < -u''(w_b - x_b),$$

which, using the first-order condition for player b, may be written as

$$\pi(x_a, x_b) \stackrel{\text{def}}{=} \frac{p_{12}(x_a, x_b) - p_{22}(x_a, x_b)}{-p_2(x_a, x_b)} x_b < A_b x_b. \tag{A.13}$$

For the power-logistic CSF, $\pi(x_a, x_b) = m\frac{1-(\frac{x_b}{x_a})^m}{1+(\frac{x_b}{x_a})^m}(1+\frac{x_b}{x_a}) - 1$.[10] Condition (A.13) compares two curvature measures. The *lhs*, $\pi(x_a, x_b)$, equals $\frac{\mathrm{d}\log\frac{\partial\Pi_b}{\partial x_b}}{\mathrm{d}\log x_b}|_{\mathrm{d}(x_a+x_b)=0}$, or the elasticity of b's marginal return to effort given that total effort remains constant. The *rhs* of (A.13) corresponds to $\frac{\mathrm{d}\log(-\frac{\partial u(w_b-x_b)}{\partial x_b})}{\mathrm{d}\log x_b}$, the elasticity of the marginal utility cost of effort. If the latter exceeds the former, b's reaction is temperate enough for aggregate effort to correlate with that of a.

What happens under uniform wealth growth? With unequal initial wealth, total effort will change with

$$(1 - f_1 g_1)(\mathrm{d}x_a + \mathrm{d}x_b) = [(1 + g_1)\, f_2 w_a + (1 + f_1)\, g_2 w_b]\,\mathrm{d}\log w, \qquad \text{(A.14)}$$

where $\mathrm{d}\log w$ denotes the common growth rate in wealth. Thus, the same sufficient condition for total effort to increase when b gets richer, ensures that total effort is a normal good. In a symmetric game, $w_a = w_b$ and therefore $x_a = x_b$, so that (A.14) reduces to

$$(\mathrm{d}x_a + \mathrm{d}x_b) = 2 f_2 w \mathrm{d}\log w > 0. \qquad \text{(A.15)}$$

We finally discuss the effects of wealth inequality. We can then invoke Theorem 2, and show the following result in A.7.

Theorem 3 *Let $A = -u''(\cdot)/u'(\cdot)$ and $P = -u'''(\cdot)/u''(\cdot)$. In the symmetric privilege contest model, the sign of the quadratic form (A.9) is positive iff*

$$2A(1 - m^2) > P. \qquad \text{(A.16)}$$

First, note that this inequality may also be written as $2A - P > 2Am^2$. Thus, if the marginal willingness to pay for rent is concave in final wealth (cf. (A.10)), a small MPS in wealth reduces total effort. When u is quadratic, $P = 0$, and the inequality reduces to $m < 1$. When u is CARA, $A = P$ and the inequality reduces to $m < 2^{-\frac{1}{2}} \simeq .707$. Thus the quadratic and CARA cases illustrate instances where the value of the decisiveness parameter of

[10] Comparing (A.13) with (A.12) shows that when the SOC for b barely holds (i.e., close to zero), a sufficiently large p_{12} will result into $1+g_1 < 0$: the increase in a's effort triggers such a strong negative reduction in b's effort that aggregate effort falls. A sufficiently large p_{12} can be provoked by both a high decisiviness parameter m and a sufficient degree of wealth inequality (small $\frac{x_b}{x_a}$, or large $\Pi_a - \Pi_b$).

the CSF determines whether the effect of a MPS in wealth on total effort is positive or negative. If we multiply (A.16) by $(w-x)$, we may replace A and P by $-\frac{u''(w-x)}{u'(w-x)}(w-x)$ and $-\frac{u'''(w-x)}{u''(w-x)}(w-x)$, the coefficients of relative risk aversion and relative prudence, respectively. When $u(\cdot)$ has constant relative-risk aversion (CRRA) denoted by ρ, the inequality reduces to $\rho(\frac{1}{2}-m^2) > \frac{1}{2}$.

A.5 Ability contest model

In this section, we demonstrate the results stated in Proposition 2 as well as some results supporting various statements regarding the effect of a MPS in wealth at the end of Section 3.

Let $U_i = \Pi_i u(w_i + r) + (1-\Pi_i)u(w_i) - c(x_i)$, with $c' > 0$ and $c'' \geq 0$. The best-response of player a, $f(x_b, w_a)$, is defined by the necessary first- and second-order conditions

$$\begin{array}{rcl} p_1(f(x_b,w_a),x_b)\Delta u_a - c'(f(x_b,w_a)) & = & 0, \\ p_{11}(f(x_b,w_a),x_b)\Delta u_a - c''(f(x_b,w_a)) & < & 0, \end{array}$$

where $\Delta u_i \stackrel{\text{def}}{=} u(w_i+r) - u(w_i) > 0$ $(i = a, b)$, and similar definitions for $\Delta u_i'$ and $\Delta u_i''$. Eliminating Δu_a from the second-order condition, this may also be written as

$$\frac{p_{11}}{p_1}x_a < \frac{c''(x_a)}{c'(x_a)}x_a. \tag{A.17}$$

Simple comparative statics show that

$$f_1 = -\frac{p_{12}\Delta u_a}{p_{11}\Delta u_a - c''(x_a)} \text{ and } f_2 = -\frac{p_1 \Delta u_a'}{p_{11}\Delta u_a - c''(x_a)} < 0,$$

where the inequality follows from the concavity of $u(\cdot)$ and the second-order condition. Similarly, player b's best-response $g(x_a, w_b)$ satisfies the necessary first- and second-order conditions

$$\begin{array}{rcl} -p_2(x_a,g(x_a,w_b))\Delta u_b - c'(g(x_a,w_b)) & = & 0, \\ -p_{22}(x_a,g(x_a,w_b))\Delta u_b - c''(g(x_a,w_b)) & < & 0, \end{array}$$

and differentiating with respect to x_a and w_b yields

$$g_1 = \frac{-p_{21}\Delta u_b}{p_{22}\Delta u_b + c''(x_b)}, \text{ and } g_2 = \frac{-p_2 \Delta u_b'}{p_{22}\Delta u_b + c''(x_b)} < 0.$$

Again, at a symmetric equilibrium, $p_{12} = 0$ and therefore $f_1 = g_1 = 0$. Hence, at a symmetric equilibrium (cf. (A.3) and (A.4))

$$\frac{\partial x_a}{\partial w_a} = f_2 < 0 = \frac{\partial x_b}{\partial w_a},$$

and Theorem 1 allows us to conclude that $x_a < x_b$ iff $w_a > w_b$.

An increase in player a's wealth reduces that player's equilibrium effort (cf. (A.3) and $f_2 < 0$). And because $p_{12} < 0$, the equilibrium effort of the poorer player, b, will also fall (cf. (A.4) and $f_2 < 0 < g_1$): that is, the poor player's effort is a strategic complement to that of the rich player. Total equilibrium effort then unambiguously declines ($\frac{f_2(1+g_1)}{1-f_1 g_1} < 0$).

Conversely, an isolated increase in the wealth of the poor player, b, reduces that player's own equilibrium effort, (cf (A.6) and $g_2 < 0$), but increases the equilibrium effort of the rich player (cf. (A.5) and $f_1, g_2 < 0$). Without further restrictions, the sign of the effect on total equilibrium effort, $\frac{g_2(1+f_1)}{1-f_1 g_1}$, is then ambiguous. Using the first-order condition for a, we show that a necessary and sufficient condition for $1 + f_1$ to be positive, i.e., for aggregate effort to fall, is

$$\frac{p_{11} - p_{12}}{p_1} x_a < \frac{c''(x_a)}{c'(x_a)} x_a. \tag{A.18}$$

Notice that $\frac{p_{11}-p_{12}}{p_1} x_a = \pi(x_b, x_a) < 0$ (where $\pi(\cdot)$ was defined in (A.13)) and we therefore obtain a similar sufficient condition as for the privilege contest model.[11] The *lhs* of (A.18) can be interpreted as $\frac{\mathrm{d}\log p_1}{\mathrm{d}\log x_a}|_{\mathrm{d}(x_a+x_b)=0}$, the elasticity of a's marginal return to effort given that total effort remains constant. The *rhs* corresponds to the elasticity of a's marginal cost of effort. If the latter elasticity exceeds the former, player a's reaction is temperate enough for aggregate effort to correlate with that of b. Observe that for $m < 1$, $\pi < 0$, (A.18) is always satisfied and aggregate effort falls when agent b alone gets wealthier.

With initially unequal wealth, general wealth growth affects total effort by (A.14), with both terms on the *rhs* negative if (A.18) holds; total effort is an inferior good. In a symmetric contest, the effect is given by (A.15) and therefore negative (as $f_2 < 0$). If (A.18) holds, a common increase in

[11] If the second-order condition for a barely holds, a reduction in x_b triggers a huge increase in x_a, resulting in a larger total effort. Both a large m or a high degree of wealth inequality ($\frac{x_a}{x_b} \ll 1$, i.e. $\Pi_b - \Pi_a \gg 0$) will contribute to this.

wealth decreases total effort in the ability contest model. With equal wealth, a common increase in wealth always decreases the efforts of both players.

We now examine the effect of a small MPS in wealth. Using Theorem 2, we can show the following result in A.7.

Theorem 4 *Consider the symmetric ability contest model with a convex power cost of effort, i.e., $c(x) = x^\gamma$ ($\gamma \geq 1$). The sign of the quadratic form (A.9) is positive iff*

$$\left(\frac{\Delta u''/\Delta u'}{\Delta u'/\Delta u}\gamma^2 - (\gamma - 1)\gamma\right) - 2m^2 > 0. \tag{A.19}$$

With CARA preferences, $A = -\frac{\Delta u''}{\Delta u'} = -\frac{\Delta u'}{\Delta u}$, and the large round bracket term simplifies to γ so that we have $m < \sqrt{\frac{\gamma}{2}}$. With quadratic preferences, $\frac{\Delta u''}{\Delta u'} = 0$, and the large round bracket term becomes $-(\gamma - 1)\gamma \leq 0$. Under CRRA, it can be shown that the term $\frac{\Delta u''/\Delta u'}{\Delta u'/\Delta u}$ has the Taylor expansion

$$\frac{1+\rho}{\rho}(1 + \frac{1}{12}\left(\frac{r}{w}\right)^2) + O(\left(\frac{r}{w}\right)^3).$$

and we have $m < \sqrt{\frac{1}{24}\frac{\rho+1}{\rho}\left(12 + (\frac{r}{w})^2\right)\gamma^2 - \frac{1}{2}(\gamma - 1)\gamma}$.

A.6 Rent-seeking contest model

In this section, we demonstrate the results stated in Proposition 3 as well as some results supporting various statements regarding the effect of a MPS in wealth at the end of Section 4.

Let $U_i = \Pi_i u(w_i + r - x_i) + (1 - \Pi_i)u(w_i - x_i)$. Using similar notations as above, a's best-response $f(x_b, w_a)$, is now defined by

$$\begin{aligned} p_1(f(x_b, w_a), x_b)\Delta u_a - Eu'_a &= 0, \\ p_{11}(f(x_b, w_a), x_b)\Delta u_a - 2p_1(f(x_b, w_a), x_b)\Delta u'_a + Eu''_a &< 0, \end{aligned}$$

where Eu'_i and Eu''_i denote expected marginal utility and its second-order equivalent ($i = a, b$). Simple computations show that

$$\begin{aligned} f_1 &= -\frac{p_{12}\Delta u_a - p_2\Delta u'_a}{p_{11}\Delta u_a - 2p_1\Delta u'_a + Eu''_a}, \text{ and} \\ f_2 &= -\frac{p_1\Delta u'_a - Eu''_a}{p_{11}\Delta u_a - 2p_1\Delta u'_a + Eu''_a}. \end{aligned} \tag{A.20}$$

Unlike the privilege and ability contest models, an increase in wealth has an ambiguous effect on the best-response function. The reason is that additional wealth reduces both the marginal benefit of winning the rent *and* the (expected) marginal cost of effort.

Similarly, player b's best-response $g(x_a, w_b)$ satisfies the necessary first- and second-order conditions

$$\begin{aligned} -p_2(x_a, g(x_a, w_b))\Delta u_b - Eu_b' &= 0, \\ -p_{22}(x_a, g(x_a, w_b))\Delta u_b + 2p_2(x_a, g(x_a, w_b))\Delta u_b' + Eu_b'' &< 0. \end{aligned}$$

Differentiating with respect to x_a and w_b, we obtain

$$\begin{aligned} g_1 &= -\frac{-p_{21}\Delta u_b + p_1 \Delta u_b'}{-p_{22}\Delta u_b + 2p_2\Delta u_b' + Eu_b''}, \text{ and} \\ g_2 &= -\frac{-p_2\Delta u_b' - Eu_b''}{-p_{22}\Delta u_b + 2p_2\Delta u_b' + Eu_b''}. \end{aligned}$$

At a symmetric equilibrium, $p_{12} = 0$, and therefore, $f_1, g_1 < 0$. Hence, at a symmetric equilibrium

$$\frac{\partial x_a}{\partial w_a} = \frac{f_2}{1 - f_1 g_1} \text{ and } \frac{\partial x_b}{\partial w_a} = \frac{g_1 f_2}{1 - f_1 g_1},$$

and we may claim that $\frac{\partial x_a}{\partial w_a}|^{\text{SE}} \gtrless 0 \gtrless \frac{\partial x_b}{\partial w_a}|^{\text{SE}}$ iff $f_2 \gtrless 0$.

Note that the sign of f_2 is given by the sign of its numerator, which upon using the first-order condition for a can be written as

$$Eu_a' \left(\frac{\Delta u_a'}{\Delta u_a} - \frac{Eu_a''}{Eu_a'} \right). \tag{A.21}$$

Let us now define two lotteries: a uniformly distributed lottery $\widetilde{z} =_d \text{U}(w_a - x_a, w_a - x_a + r)$ and a binary lottery $\widetilde{y} =_d (w_a - x_a + r, \frac{1}{2}; w_a - x_a, \frac{1}{2})$, so that the term in round brackets can be written as[12]

$$\frac{-Eu_a''(\widetilde{y})}{Eu_a'(\widetilde{y})} - \frac{-Eu_a''(\widetilde{z})}{Eu_a'(\widetilde{z})}.$$

Given that the binary lottery $(\widetilde{y})$ is a MPS of the uniform lottery $(\widetilde{z})$, the sign of f_2 is positive (resp. negative) if the MPS of a background risk increases (resp. decreases) the coefficient of absolute risk aversion.

[12] $E_U u'(\widetilde{z}) = \int_{w_a - x_a}^{w_a - x_a + r} u_a'(\widetilde{z})\frac{1}{r}\mathrm{d}\widetilde{z} = \frac{1}{r}\Delta u_a$ and $E_U u''(\widetilde{z}) = \int_{w_a - x_a}^{w_a - x_a + r} u_a''(\widetilde{z})\frac{1}{r}\mathrm{d}\widetilde{z} = \frac{1}{r}\Delta u_a'$.

Following Definition 1, if $u \in \Omega$, then $\frac{\partial x_a}{\partial w_a} > 0 > \frac{\partial x_b}{\partial w_a}$ at a symmetric equilibrium as $g_1 < 0$. Hence, Theorem 1 allows us to conclude that for $u \in \Omega$, in an asymmetric rent-seeking game $w_a > w_b$ implies $x_a > x_b$, and therefore $p_{12} > 0$. As a result, $u \in \Omega$ ensures that an isolated increase in a's wealth will raise that player's equilibrium effort level. The equilibrium reaction of the poorer agent, b, is negative. As before, aggregate effort will increase iff $1 + g_1 > 0$. For the rent-seeking contest model, this condition is equivalent to

$$\frac{p_{21} - p_{22}}{-p_2} + \frac{2p_2 - p_1}{-p_2}\frac{\Delta u_b'}{\Delta u_b} + \frac{Eu_b''}{Eu_b'} < 0$$

$$\Updownarrow \ \frac{p_1}{p_2} = \frac{x_b}{x_a}$$

$$\pi(x_a, x_b) + \left(1 + \frac{x_b}{x_a}\right)\left(-\frac{\Delta u_b'}{\Delta u_b}\right)x_b < \left[\left(-\frac{Eu_b''}{Eu_b'}\right) - \left(-\frac{\Delta u_b'}{\Delta u_b}\right)\right]x_b. \quad \text{(A.22)}$$

We know that the *rhs* is positive if $u \in \Omega$. But as the second *lhs* term is positive, $\pi(x_a, x_b) < 0$ is no longer sufficient for $1 + g_1 > 0$.

If the poor person becomes wealthier, that player's effort changes with $\frac{g_2}{1-f_1 g_1}$, which is positive if $u \in \Omega$ (the reasoning is the same as for f_2). The rich agent's equilibrium effort changes with $\frac{f_1 g_2}{1-f_1 g_1}$. From (A.20), it transpires that $f_1 > 0$ iff $\frac{p_{12}}{-p_2} > -\frac{\Delta u_a'}{\Delta u_a}$. Since the sign of p_{12} depends on that of $x_a - x_b$, a necessary condition for a to increase effort is that a is sufficiently richer than b. As b's wealth approaches that of a, the latter will begin to reduce effort despite the fact that b is increasing effort. The two effort levels then turn into strategic substitutes. Thus, in the rent-seeking contest model, the nature of the strategic interaction depends on wealth levels. This possibility of strategic substitutability also blurs the effect of w_b on aggregate effort. Indeed, a similar argument as above shows that $1 + f_1 > 0$ iff

$$\frac{p_{11} - p_{12}}{p_1} - \frac{2p_1 - p_2}{p_1}\frac{\Delta u_a'}{\Delta u_a} + \frac{Eu_a''}{Eu_a'} < 0$$

$$\Updownarrow$$

$$\pi(x_b, x_a) + \left(1 + \frac{x_a}{x_b}\right)\left(-\frac{\Delta u_a'}{\Delta u_a}\right)x_a < \left[\left(-\frac{Eu_a''}{Eu_a'}\right) - \left(-\frac{\Delta u_a'}{\Delta u_a}\right)\right]x_a. \quad \text{(A.23)}$$

Given $\frac{x_a}{x_b} > 1$, the first *lhs* term is negative (since $p_{12} > 0 > p_{11}$). The *rhs* is positive if $u \in \Omega$. Once again, the positive second *lhs* term blurs the inequality.

The CARA utility function satisfies the conditions for Ω "just" (since background risk has no effect on absolute risk aversion under CARA). Hence, it provides a boundary case where $f_2 = 0$ and $g_2 = 0$, which is easily checked as both $\frac{-\Delta u_i'}{\Delta u_i}$ and $\frac{-Eu_i''}{Eu_i'}$ equal the coefficient of absolute risk aversion. The quadratic utility function provides another case where $f_2 = 0$ and $g_2 = 0$. In both cases, aggregate effort is unaffected by an isolated increase in wealth.

With a common increase in wealth, aggregate efforts change with $2\frac{f_2}{1-f_1}$. Hence, $u \in \Omega$ ensures that uniform growth in wealth will increase the representative agent's effort.

We finally discuss the effect of wealth inequality on aggregate effort in the rent-seeking contest model. For the reason discussed earlier, there is no effect of wealth distribution across players under CARA or quadratic utility. The following theorem is proven in A.7.

Theorem 5 *In the symmetric rent-seeking contest model, the sign of the quadratic form (A.9) is positive iff*

$$p_{11}^2 p_1 T_1 + 4p_{11}p_1^2 T_2 + (3p_{112} - p_{111})\, p_1^2 T_3 + 4p_1^3 T_4 < 0 \tag{A.24}$$

where

$$\begin{aligned}
T_1 &= \frac{Eu''}{Eu'}\left(\frac{Eu'''}{Eu''} - 2\frac{\Delta u'}{\Delta u}\right) - \frac{\Delta u'}{\Delta u}\left(\frac{\Delta u''}{\Delta u'} - 2\frac{\Delta u'}{\Delta u}\right), \\
T_2 &= \frac{\Delta u'}{\Delta u}\left[\frac{\Delta u'}{\Delta u}\left(\frac{Eu''}{Eu'} - \frac{\Delta u'}{\Delta u}\right) - \frac{Eu''}{Eu'}\left(\frac{Eu'''}{Eu''} - \frac{\Delta u''}{\Delta u'}\right)\right], \\
T_3 &= \left(\frac{Eu''}{Eu'} - \frac{\Delta u'}{\Delta u}\right)^2, \\
T_4 &= \left(\frac{\Delta u'}{\Delta u}\right)^2 \left[\frac{\Delta u''}{\Delta u'}\left(\frac{\Delta u'}{\Delta u} - \frac{Eu''}{Eu'}\right) + \frac{Eu''}{Eu'}\left(\frac{Eu'''}{Eu''} - \frac{\Delta u''}{\Delta u'}\right)\right].
\end{aligned}$$

With CARA preferences, all ratios in T_1, T_2, T_3, and T_4 coincide with $-A$, and therefore the four terms equal zero. The same is true for quadratic preferences, $u(y) = y - \frac{\beta}{2}y^2$.[13] In A.7 below, we can then prove the following theorem.

[13]In that case, $\Delta u = \Delta y(1 - \beta Ey)$, $\Delta u' = -\beta \Delta y$, $\Delta u'' = 0$, $Eu' = 1 - \beta Ey$, $Eu'' = -\beta, Eu''' = 0$, where $\Delta y = r$ and $Ey = w - x + \frac{1}{2}r$. Then $\frac{\Delta u'}{\Delta u} = \frac{Eu''}{Eu'} = -\frac{\beta}{1-\beta Ey}$ and $\frac{\Delta u''}{\Delta u'} = \frac{Eu'''}{Eu''} = 0$. Once again, all four terms vanish.

Theorem 6 *With CRRA preferences and small $\frac{r}{w}$, the inequality (A.24) is violated.*

None of the preferences considered in the results above (i.e., quadratic, CARA, and CRRA) result in larger aggregate efforts. At the same time, these three types of preferences share a non-negative third derivative of $u(\cdot)$ ("prudence"). This suggests that a negative third derivative ("imprudence") may be a necessary condition for a MPS in wealth to raise aggregate effort. This conjecture is supported by the following example.

Example 1 *Suppose that $u(y) = y - \frac{\beta}{2}y^2 + \frac{\gamma}{3}y^3$ with $\beta = \frac{1}{15}$ and $\gamma \leq \frac{2}{1000}$, such that $u''(y) < 0$ for all $y < 15$. Then for a symmetric rent-seeking contest model with $w = 10, r = 1, m = 1.5$, and for $\gamma \in [-.002, 0]$, a small MPS in wealth results in higher aggregate efforts, as shown in Table A1.*

Table A1. Results for cubic preferences.[a]

γ	x^*	SOC	$Eu(x^*) - Eu(0)$	qf
-.0020	.3760	-.2114	.0023	.0052
-.0015	.3755	-.3583	.0099	.0006
-.0010	.3753	-.5055	.0176	.0001
-.0005	.3751	-.6527	.0252	.00002
0	.375	-.8	.0328	0
.0005	.3749	-.9473	.0404	-.000004
.0010	.3749	-1.0947	.0480	-.000003
.0015	.3748	-1.2420	.0557	-.000001

[a]The columns respectively provide the values of the "prudence coefficient" γ, the equilibrium effort (x^*), the value of the second-order condition (SOC), the gain in expected utility when playing x^* rather than 0 against x^*, and the value of the lhs of (A.24) (qf).

A.7 Proofs of Theorems 3, 4, 5 and 6

For all three contest models, we can say that the first- and second-order conditions for agent a are given by

$$\begin{aligned} h(x_a, w_a, x_b) &= 0, \\ h_1(x_a, w_a, x_b) &< 0. \end{aligned}$$

Hence, the optimal responses to $\mathrm{d}w_a$ and $\mathrm{d}x_b$ are given by

$$\frac{\partial x_a}{\partial w_a} = -\frac{h_2}{h_1} \text{ and } \frac{\partial x_a}{\partial x_b} = -\frac{h_3}{h_1}.$$

The second-order responses are then given by

$$\begin{aligned}
\frac{\partial^2 x_a}{\partial w_a^2} &= \frac{\partial(-\frac{h_2}{h_1})}{\partial w_a} + \frac{\partial(-\frac{h_2}{h_1})}{\partial x_a}\frac{\partial x_a}{\partial w_a} \\
&= \left(-\frac{1}{h_1}\right)\left[h_{22} - 2\frac{h_2}{h_1}h_{12} + \left(\frac{h_2}{h_1}\right)^2 h_{11}\right] \qquad \text{(A.25)} \\
\frac{\partial^2 x_a}{\partial w_a \partial x_b} &= \frac{\partial(-\frac{h_2}{h_1})}{\partial x_b} + \frac{\partial(-\frac{h_2}{h_1})}{\partial x_a}\frac{\partial x_a}{\partial x_b} \\
&= \left(-\frac{1}{h_1}\right)\left[h_{23} - \frac{h_2}{h_1}h_{13} - \frac{h_3}{h_1}h_{21} + \frac{h_2}{h_1}\frac{h_3}{h_1}h_{11}\right] \qquad \text{(A.26)} \\
\frac{\partial^2 x_a}{\partial x_b^2} &= \frac{\partial(-\frac{h_2}{h_1})}{\partial x_b} + \frac{\partial(-\frac{h_2}{h_1})}{\partial x_a}\frac{\partial x_a}{\partial x_b} \\
&= \left(-\frac{1}{h_1}\right)\left[h_{33} - 2\frac{h_3}{h_1}h_{13} + \left(\frac{h_3}{h_1}\right)^2 h_{11}\right]. \qquad \text{(A.27)}
\end{aligned}$$

Proof of Theorem 3

For the privilege contest model, we have the following h-functions:

$$\begin{aligned}
h &= -u'(w_a - x_a) + p_1 r = 0 \\
h_1 &= u''(w_a - x_a) + p_{11} r < 0 \\
h_2 &= -u_a'', h_3 = p_{12} r \overset{\text{SE}}{=} 0, \\
h_{11} &= -p_{111} r - u_a''', h_{12} = u_a''', h_{13} = p_{112} r, \\
h_{22} &= -u_a''', h_{23} = 0, h_{33} = p_{122} r.
\end{aligned}$$

With the help of (A.25)-(A.27), the partials of $f(x_b, w_a)$ can then be computed

$$\begin{aligned}
f_{11} &= \left(-\frac{r}{h_1^3}\right)\left(-u_a''' p_{11}^2 r + (u_a'')^2 p_{111}\right), \\
f_{22} &= \left(-\frac{r}{h_1^3}\right) p_{122} h_1^2, \\
f_{12} &= \left(-\frac{r}{h_1^3}\right) u_a'' p_{112}\left(u_a'' + p_{11} r\right).
\end{aligned}$$

Given $h_3 = 0$, we obtain that $f_1 = 0$ and $1 + f_1 = 1$. Applying Theorem 2 then obtains that the sign of the quadratic form (A.9) is given by the sign of

$$A\left(p_{111} - 3p_{112}\right) - P\frac{p_{11}^2}{p_1},$$

where $A \stackrel{\text{def}}{=} -\frac{u''(w-x)}{u'(w-x)}$ and $P \stackrel{\text{def}}{=} -\frac{u'''(w-x)}{u''(w-x)}$. Making use of the expressions for the p-derivatives gives

$$A\frac{1}{2}\frac{m}{x_3}\left(1 - m^2\right) - P\frac{1}{4}\frac{m}{x_3}.$$

This proves Theorem 3.

Proof of Theorem 4

For the ability contest model, we obtain the following expressions for the h function

$$\begin{aligned}
h &= \frac{\partial p}{\partial x_a}\Delta u_a\left[u(w_a + r) - u(w_a)\right] - c_a' = 0 \\
h_1 &= \frac{\partial^2 p}{\partial x_a^2}\Delta u_a - c_a'' < 0 \\
h_2 &= \frac{\partial p}{\partial x_a}\Delta u_a', h_3 = \frac{\partial^2 p}{\partial x_a \partial x_b}\Delta u_a \stackrel{\text{SE}}{=} 0 \\
h_{11} &= \frac{\partial^3 p}{\partial x_a^3}\Delta u_a - c_a''', h_{12} = \frac{\partial^2 p}{\partial x_a^2}\Delta u_a' \\
h_{13} &= \frac{\partial^3 p}{\partial x_a^2 \partial x_b}\Delta u_a, h_{22} = \frac{\partial p}{\partial x_a}\Delta u_a'' \\
h_{23} &= \frac{\partial^2 p}{\partial x_a \partial x_b}\Delta u_a' \stackrel{\text{SE}}{=} 0, h_{33} = \frac{\partial^3 p}{\partial x_a \partial x_b^2}\Delta u_a,
\end{aligned}$$

where $\Delta u_a \stackrel{\text{def}}{=} [u(w_a + r) - u(w_a)]$. Making use of (A.25)-(A.27), we obtain the following curvatures for the best-response function:

$$\begin{aligned}
f_{22} &= \left(-\frac{1}{h_1^3}\right)\left(p_1 \Delta u_a'' h_1^2 - 2p_1 p_{11}(\Delta u_a')^2 h_1 + p_1^2(\Delta u_a')^2(p_{111}\Delta u_a - c''')\right), \\
f_{11} &= \left(-\frac{1}{h_1^3}\right)p_{122}\Delta u_a h_1^2, \\
f_{12} &= \left(-\frac{1}{h_1^3}\right)\left(-p_{112}p_1 \Delta u_a' \Delta u_a h_1\right).
\end{aligned}$$

Given $h_3 = 0$, $f_1 = 0$ and $1 + f_1 = 1$, application of Theorem 1 gives that the quadratic form (A.9) is given by

$$\left(-\frac{1}{h_1^3}\right)\left\{-3(\Delta u')^2 \Delta u p_1^2 p_{112} + [\Delta u'' \Delta u - 2(\Delta u')^2]\Delta u p_1 (p_{11})^2 \right.$$
$$+(\Delta u')^2 \Delta u p_1^2 p_{111} - 2[\Delta u'' \Delta u - (\Delta u')^2] p_1 p_{11} c'' + \Delta u'' p_1 c''^2$$
$$\left. -(\Delta u')^2 p_1^2 c''' \right\}.$$

Since $h_1 < 0$ (SOC), the sign of the quadratic form is the sign of the term in curly brackets. Making use of the derivatives of the power-logistic CSF in the symmetric equilibrium, the power specification for $c(x)$, $c(x) = x^\gamma$, and the FOC evaluated at the symmetric equilibrium, $\frac{m}{4x}\Delta u = c'$, this term can be written as (up to a positive constant)

$$\left(-2m^2 + \frac{\Delta u''/\Delta u'}{\Delta u'/\Delta u}\right) + (\gamma - 1)\left(\frac{\Delta u''/\Delta u'}{\Delta u'/\Delta u}(1+\gamma) - \gamma\right).$$

Then the quadratic form is therefore positive iff

$$\frac{\Delta u''/\Delta u'}{\Delta u'/\Delta u}\gamma^2 - (\gamma - 1)\gamma > 2m^2.$$

This proves Theorem 4.

Proof of Theorem 5

For the rent-seeking contest model, the h-functions are given by

$$\begin{aligned}
h &= p_1 \Delta u_a - E u_a' = 0,\\
h_1 &= p_{11}\Delta u_a - 2p_1 \Delta u_a' + E u_a'' < 0\\
h_2 &= p_1 \Delta u_a' - E u_a'', h_3 = p_{12}\Delta u_a - p_2 \Delta u_a' \overset{\text{SE}}{=} p_1 \Delta u_a'\\
h_{11} &= p_{111}\Delta u_a - 3p_{11}\Delta u_a' + 3p_1 \Delta u_a'' - E u_a'''\\
h_{12} &= p_{11}\Delta u_a' - 2p_1 \Delta u_a'' + E u_a'''\\
h_{13} &= p_{112}\Delta u_a - 2p_{12}\Delta u_a' + p_2 \Delta u_a'' \overset{\text{SE}}{=} p_{112}\Delta u_a - p_1 \Delta u_a''\\
h_{22} &= p_1 \Delta u_a'' - E u_a''', h_{23} = p_{12}\Delta u_a' - p_2 \Delta u_a''\\
h_{33} &= p_{122}\Delta u_a - p_{22}\Delta u_a' \overset{\text{SE}}{=} -p_{112}\Delta u_a + p_{11}\Delta u_a',
\end{aligned}$$

where $\Delta u_a \overset{\text{def}}{=} u(w_a + r - x_a) - u(w_a - x_a)$. With the help of these derivatives and expressions (A.25)-(A.27), the curvatures f_{11}, f_{12}, and f_{22} for a's best-response function are computed. Using Theorem 2, and simple, but tedious, factorization, it can be shown[14] that the sign of the quadratic form $F_{11} - 2F_{12} + F_{22}$ can be written as

$$\frac{1}{G}\left[p_{11}^2 p_1 T_1 + 4p_{11}p_1^2 T_2 + (3p_{112} - p_{111})\, p_1^2 T_3 + 4p_1^3 T_4\right], \tag{A.28}$$

where

$$\begin{aligned}
T_1 &= \frac{Eu''}{Eu'}\left(\frac{Eu'''}{Eu''} - 2\frac{\Delta u'}{\Delta u}\right) - \frac{\Delta u'}{\Delta u}\left(\frac{\Delta u''}{\Delta u'} - 2\frac{\Delta u'}{\Delta u}\right),\\
T_2 &= \frac{\Delta u'}{\Delta u}\left[\frac{\Delta u'}{\Delta u}\left(\frac{Eu''}{Eu'} - \frac{\Delta u'}{\Delta u}\right) - \frac{Eu''}{Eu'}\left(\frac{Eu'''}{Eu''} - \frac{\Delta u''}{\Delta u'}\right)\right],\\
T_3 &= \left(\frac{Eu''}{Eu'} - \frac{\Delta u'}{\Delta u}\right)^2,\\
T_4 &= \left(\frac{\Delta u'}{\Delta u}\right)^2\left[\frac{\Delta u''}{\Delta u'}\left(\frac{\Delta u'}{\Delta u} - \frac{Eu''}{Eu'}\right) + \frac{Eu''}{Eu'}\left(\frac{Eu'''}{Eu''} - \frac{\Delta u''}{\Delta u'}\right)\right], \text{ and}\\
G &= \left(p_{11} - p_1\frac{\Delta u'}{\Delta u} + p_1\frac{Eu''}{Eu'}\right)\left(p_{11} - 3p_1\frac{\Delta u'}{\Delta u} + p_1\frac{Eu''}{Eu'}\right)^2.
\end{aligned}$$

Note that G is negative given the term in the first round brackets can be written as $\frac{h_1}{\Delta u} + \frac{\partial p_a}{\partial x_a}\frac{\Delta u'}{\Delta u}$ and both h_1 and $\Delta u'$ are negative because of the second-order condition and risk aversion, respectively. This proves Theorem 5.

Proof of Theorem 6

The first-order condition for x is given by $h(x_a, w_a, x_b) = 0$, where

$$\begin{aligned}
h(x_a, w_a, x_b) = {}& \frac{\partial p_a(x_a, x_b)}{\partial x_a}[u(w_a + r - x_a) - u(w_a + r - x_a)]\\
&-[p(x_a, x_b)u(w_a + r - x_a) + (1 - p(x_a, x_b))u(w_a - x_a)].
\end{aligned}$$

At a symmetric equilibrium ($x = x_a = x_b$), $\frac{\partial p_a(x,x)}{\partial x_a} = \frac{1}{4}\frac{m}{x}$ and $p_a(x,x) = \frac{1}{2}$. Using for $u(y)$ the CRRA form, $u(y) = \frac{y^{1-\rho}}{1-\rho}$, and taking a Taylor expansion of degree 2 around $r = 0$, results in

$$h(x, w, x) \simeq (w - x)^{-\rho}\left[-1 + \left(\frac{1}{4}\frac{m}{x} + \frac{1}{2}\frac{\rho}{w - x}\right) r - \left(\frac{1}{8}\frac{m}{x}\frac{\rho}{w - x} + \frac{1}{4}\frac{\rho(1 + \rho)}{(w - x)^2}\right) r^2\right].$$

[14] The Maple files are available from the authors upon request.

Equating the *rhs* to zero and solving for x gives three roots, with the real solution being $\frac{1}{4}mr+O(r^3)$ (in fact $\frac{1}{4}mr$ is the solution to the case of quadratic preferences). Replacing x by $\frac{1}{4}mr$, the obtained expressions for T_1, T_2, T_3, and T_4 are then Taylor-approximated around $r = 0$:

$$
\begin{aligned}
T_1 &= \frac{1}{2}\frac{\rho(1+\rho)}{w^4}r^2 + O(r^3), \\
T_2 &= \frac{1}{3}\frac{\rho^2(1+\rho)}{w^5}r^2 + O(r^3), \\
T_3 &= O(r^3), \text{ and} \\
T_4 &= \frac{2}{3}\frac{\rho^3(1+\rho)}{w^6}r^2 + O(r^3).
\end{aligned}
$$

Next, the coefficients with T_1, T_2, and T_4 are computed using the earlier derived expressions for the probability function and its derivatives, and evaluating them at $x = \frac{1}{4}mr$. Finally, the numerator of (A.28) is computed. Up to a negative proportionality factor, it is equivalent to

$$
1 - \frac{2}{3}\left(\rho m \frac{r}{w}\right) + \frac{1}{2}\left(\rho m \frac{r}{w}\right)^2.
$$

The expression has no real roots and is always positive. Hence, for CRRA preferences and a rent that is small w.r.t. the initial wealth w, a small MPS in wealth reduces aggregate effort. This proves Theorem 6.